Land Development:
Risk, Return and Risk Management

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Abstract

Real estate development is a multiphase process involving land (horizontal) development, followed by residential and/or commercial (vertical) development, ending with the eventual marketing phase of the development through the sale or leasing of the completed site. For example, the residential production cycle begins when a land developer purchases a tract of land, receives appropriate regulatory approvals, constructs needed infrastructure over time, and divides the larger parcel into multiple lots. Typically, the land developer then sells the finished lots to a third party homebuilder. After the homebuilder completes construction, the housing units are generally sold in the owner-occupied housing market. While all three phases of the housing industry are interrelated, each stage involves various risks which are allocated between landowners, land developers, and the homebuilders.

In this paper we model, using contingent claims valuation and risk-neutral pricing methodology, the entire development process including the debt financing of land acquisition and construction expenditures. We are able to determine endogenously both the land valuation and debt pricing in the presence of bankruptcy costs. This allows us to determine spreads on debt given a certain level of presales and to also calculate expected returns to equity for various levels of presales and debt financing. Our results are able to justify the high equity return expectations that are revealed in the market surveys. We also model the presale option value and the common practice of applying the option premium (or binder as it is referred to by the industry) as a reduction to the exercise price.

I. Introduction

The real estate development industry can often be described as a multiphase process involving land (horizontal) development, followed by residential and/or commercial (vertical) development, ending with the eventual marketing phase of the development through the sale or leasing of the completed site. For example, the residential production cycle begins when a land developer purchases a tract of land, receives appropriate regulatory approvals, constructs needed infrastructure over time, and divides the larger parcel into multiple lots. Typically, the land developer then sells the finished lots to a third party homebuilder. After the homebuilder completes construction, the housing units are generally sold in the owner-occupied housing market. While all three phases of the housing industry are interrelated, each stage involves various risks which are allocated between landowners, land developers, and the homebuilders.

The land development phase is often identified as one of the riskiest phases of the real estate production process. Often, the developer must acquire land, expend upfront time and money in the regulatory process, and invest in needed infrastructure with uncertain costs before generating any positive cash flows.[1] One major risk of land development is reflected in the price volatility of the completed and subdivided land prices, which are primarily affected by market demand. Completed lot prices tend to vary more than price levels for completed building projects and time to build exacerbates the range of sale prices and timing of the sales that may be received. Since the value of a subdivision is determined by lot prices and the pace in which the lots are absorbed in the market, changing economic conditions, consumer preferences, and increasing

[1] Risk in the regulatory approval process often involves taking a parcel of land through the rezoning process, obtaining site and subdivision approval, and also obtaining construction permits. All of these steps must be complete before raw land development is allowed to move forward.

competition are all critical concerns. If prices and absorption rates are below expectations, the developer is exposed to significant downside risk.

The risks inherent in land development can be directly observed in the return expectations of land developers. Surveys examining return requirements in the land development industry are limited, but Owens (1998) suggests land developers may require returns as much as 30% higher than those of residential homebuilders.[2] Similar and more recent surveys done by Korpacz/PriceWaterhouseCoopers find comparable, although somewhat lower return expectations.[3]

Presales are common strategies used to reduce the developer's exposure to downturns in market demand for completed lots. Developers use this risk management technique in the land development process to reduce or shift the risk to the eventual purchaser of the completed lots, often the vertical developer. Many developers and their financing institutions find presales an effective method to reduce marketing risk.

Presales in the residential development market involve the conditional sale of lots through options to third party homebuilders before the subdivision is completed. Homebuilders are often willing to purchase lots offered in a presale because it provides an opportunity to lock a fixed price in anticipation of increasing land values or increased competition for sites. The presale option is priced such that it rewards the homebuilder for sharing the risk of future demand volatility.[4]

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[2] Developer surveys conducted by Owens found internal rate of return requirements of 10% for residential homebuilders, 15-20% for development of zoned land, and 25-40% for development of raw land. Note that these returns are based a high degree of leverage which is commonly used in al phases of the development process.

[3]The slightly lower return expectations observed recently are due to the current low return requirements for all alternative investments driven by the current low interest rate environment

[4] Empirical research conducted by Sirmans, Turnbull and Dombrow (1997) suggests the discounted presale price is appropriate due to first mover disadvantages associated with being one of the first consumers to purchase a lot within a development. The first builder to purchase a lot in the subdivision does not know

Lai, Wong, and Zhou (2004) identify that presales option contracts provide a benefit to limit risk because they allow the developer to commence construction, while limiting inventory costs and bankruptcy risk and reducing uncertainty about future demand.[5] They model the value of the presale contract and show that, in the presence of risk-neutral buyers and risk-averse developers, it is always optimal for a developer to presell units to mitigate price risk.

While the Lai, Wong, and Zhou paper provides an initial analysis f the risk management technique of presales, there are many unanswered questions to be addressed. In this paper, we also model the presale option value and the common practice of applying the option premium (or binder as it is referred to by the industry) as a reduction to the exercise price. More importantly we model, using contingent claims valuation and risk-neutral pricing methodology, the entire development process including the debt financing of land acquisition and construction expenditures. We are able to determine endogenously both the land valuation and debt pricing in the presence of bankruptcy costs. This allows us to determine spreads on debt given a certain level of presales and to also calculate expected returns to equity for various levels of presales and debt financing. Our results are able to justify the high equity return expectations that are revealed in the market surveys.

We proceed as follows. In the next section we present the general economic environment of the development process and develop the financial model. In Section III

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with certainty how the neighborhood will evolve over time and must be offered a price discount to compensate for this first mover disadvantage. As the neighborhood develops, future purchasers pay higher prices for finished lots because they are provided with more information regarding the neighborhood's characteristics.

[5] Bankruptcy costs and the use of debt are not actually modeled in their paper.

we present solutions to the model for a variety of economic environments and parameter values and discuss the implications of these results. Finally, in section IV we discuss our conclusions.

II. The Model

1. Basic Assumptions

A developer purchases vacant land at time T_0 for a competitively determined price L. We assume that the land is ripe for development and that construction commences immediately, requiring total construction outlays of K. Construction time, T_C, is known with certainty at time T_0, so the project will be complete at time $T = T_0 + T_C$.

The developer either sells lots to homebuilders at time T at the market price $P(T)$, or presales lots to homebuilders at time T_0 using option contracts. In our model, the down payment (or binder as it is called in the industry) is the option premium and is specified as a percentage of the current lot value. The down payment is applied to the purchase price (strike price) when the lot is complete. The strike price is allowed to increase deterministically over time. At the time of lot completion, T, the homebuilder who purchased lots under this option contract has the right, but not the obligation to "take-down" the lots at the strike price. As with any option, they can choose to walk away.

Let $(\Omega, F, \{F_t\}_{t\geq 0}, \mathrm{P})$ be a filtered probability space. We define a market environment consisting of a risky asset representing lot prices P and a risk-free bond B.

The prices of these two assets are assumed to be governed by the stochastic differential equation,

$$dP(t) = (\mu - \delta)P(t)\,dt + \sigma P(t)\,dW(t), \quad t \in [0,T], \tag{1}$$

where $\mu \in R$ is the drift of the completed lot price over time, $\sigma \in R$ is the constant volatility rate, W is a standard Brownian motion under P, $\delta \in R^{+}$ is the convenience yield and

$$dB(t) = r\,dt, \quad t \in [0,T], \tag{2}$$

where $r \in R^{+}$ is the risk free rate of return.

As is well-known, there are no arbitrage opportunities in such a market environment if and only if there exists a probability measure $\tilde{\mathrm{P}}$ under which $P(t)e^{\delta t}/B(t)$ follows a martingale. On the probability space $(\Omega, F, \{F_t\}_{t\geq 0}, \tilde{\mathrm{P}})$ equation (1) becomes

$$dP(t) = (r - \delta)P(t)\,dt + \sigma P(t)\,d\tilde{W}(t), \quad t \in [0,T], \tag{3}$$

where, for all $t \in [0,T]$

$$\tilde{W}(t) = W(t) + \int_0^t \frac{\mu - r}{\sigma}\,du \tag{4}$$

is a standard Brownian motion under $\tilde{\mathrm{P}}$.

2. Pricing Presale Options

The presale option differs from a standard call option in that the option premium is applied to the purchase price of the lot. Thus, if $V(T_0, P(T_0); T_C)$ denotes the time T_0 no-arbitrage price of a European presale option expiring at time $T = T_0 + T_C$, then the value of a presale option to a builder at time T is

$$[P(T) - (X(T) - V(T_0, P(T_0); T_C))]^{+},$$

where

$$V(T_0, P(T_0); T_C) = e^{-rT_C} \tilde{E}\left\{[P(T) - (X(T) - V(T_0, P(T_0); T_C))]^+ \mid F(T_0)\right\} \tag{5}$$

Evaluating the conditional expectation in (5), we have that V satisfies

$$V(T_0, P(T_0); T_C) = e^{-\delta T_C} P(T_0) \frac{N(d_1)}{1 - e^{-rT_C} N(d_2)} - e^{-rT_C} X(T) \frac{N(d_2)}{1 - e^{-rT_C} N(d_2)} \tag{6}$$

where

$$d_1 = \frac{\ln(P(T_0)) - \ln(X(T) - V(T_0, P(T_0); T_C)) + (r - \delta + \sigma^2/2)T_C}{\sigma\sqrt{T_C}}, \tag{7}$$

$$d_2 = \frac{\ln(P(T_0)) - \ln(X(T) - V(T_0, P(T_0); T_C)) + (r - \delta - \sigma^2/2)T_C}{\sigma\sqrt{T_C}}, \tag{8}$$

and $N(\cdot)$ denotes the cumulative normal density function. Equation (6) can be solved for V using straightforward numerical methods.

If the price of the presale option is quoted in terms of a percentage y of the lot value, then from (6) we have

$$y = \frac{e^{-\delta T_C} P(T_0) N(d_1) - e^{-rT_C} X(T) N(d_2)}{P(T_0)(1 - e^{-rT_C} N(d_2))}. \tag{9}$$

where

$$d_1 = \frac{\ln(P(T_0)) - \ln(X(T) - yP(T_0)) + (r - \delta + \sigma^2/2)T_C}{\sigma\sqrt{T_C}} \tag{10}$$

$$d_2 = \frac{\ln(P(T_0)) - \ln(X(T) - yP(T_0)) + (r - \delta - \sigma^2/2)T_C}{\sigma\sqrt{T_C}}. \tag{11}$$

One interesting implication is that increases in the risk-neutral probability at time T_0 that the presale option will expire in-the-money lead to increases in y thereby shifting more price risk from the developer to the builder (option purchaser).

3. Return to the Builder

Using real probabilities, the expected payoff of the option to the builder is

$$\begin{aligned} &E\left\{[P(T)-(X(T)-V(T_0,P(T_0);T_C))]^+ \mid F(T_0)\right\} \\ &= P(T_0)e^{(\mu-\delta)T_C}N(d_1^*)-(X(T)-yP(T_0))N(d_2^*) \end{aligned} \tag{12}$$

where

$$d_1^* = \frac{\ln(P(T_0))-\ln(X(T)-yP(T_0))+(\mu-\delta+\sigma^2/2)T_C}{\sigma\sqrt{T_C}} \tag{13}$$

$$d_2^* = \frac{\ln(P(T_0))-\ln(X(T)-yP(T_0))+(\mu-\delta-\sigma^2/2)T_C}{\sigma\sqrt{T_C}}. \tag{14}$$

So, using real probabilities, the expected return of the option to the builder is

$$\begin{aligned} &\frac{P(T_0)e^{(\mu-\delta)T_C}N(d_1^*)-(X(T)-yP(T_0))N(d_2^*)}{yP(T_0)}-1 \\ &= \frac{e^{(\mu-\delta)T_C}}{y}N(d_1^*)-N(-d_2^*)-\frac{X(T)}{yP(T_0)}N(d_2^*) \end{aligned}$$

Assuming that all in-the-money presale options will be exercised, then the probability (using real probabilities) that a presale option is exercised is

$$
\begin{aligned}
&P\{P(T) > X(T) - yP(T_0)\} \\
&= P\left\{W(T_C) > \frac{1}{\sigma}\left[\ln\left(\frac{X(T)}{P(T_0)} - y\right) - (\mu - \delta - \sigma^2/2)T_C\right]\right\} \\
&= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d_2^*} e^{\frac{-z^2}{2}}\, dz \\
&= N(d_2^*),
\end{aligned}
$$

4. Return to the Developer

We assume that the developer purchases land at time T_0 for L and divides it into M indistinguishable lots (M is arbitrary, but once chosen remains fixed). The developer has a number of lots m ($0 \le m \le M$) that are pressold with options. The value of the development to the developer at time T is given by the $F(T)$-measurable random variable

$$
(M - m)P(T) + mP(T)1_{\{P(T) \le X(T) - yP(T_0)\}} + m(X(T) - yP(T_0))1_{\{P(T) > X(T) - yP(T_0)\}},
$$

where 1_A denotes the indicator function on an event A. Let V_t denote the value of the development to the developer at time $t \in [T_0, T]$. Then the time T_0 expected value (using real probabilities) of the development to the developer at time T is

$$
\begin{aligned}
E[V_T \mid F(T_0)] &= E[(M - m)P(T) + m(X(T) - yP(T_0))1_{\{P(T) > X(T) - yP(T_0)\}} + mP(T)1_{\{P(T) \le X(T) - yP(T_0)\}} \mid F(T_0)] \\
&= (M - m + mN(-d_1^*))P(T_0)e^{(\mu-\delta)T_C} + m(X(T) - yP(T_0))N(d_2^*) \\
&= (M - mN(d_1^*))P(T_0)e^{(\mu-\delta)T_C} + m(X(T) - yP(T_0))N(d_2^*).
\end{aligned} \tag{15}
$$

The developer must spend a total amount K for construction over the time interval $[T_0, T]$. Let $k(t)$ denote the rate of spending by the developer on construction at time t. We assume that construction outlays are made at a constant rate, so that $dk(t) = K/T_C dt$.

5. All Equity Expected Project IRR

If the developer finances the project entirely with equity, then the expected (internal rate of return (IRR)) for the project, r^*, satisfies

$$\begin{aligned} e^{-r^* T_C} E_{T_0}\left(V_T\right) &= (L - myP(T_0)) + \int_{T_0}^{T} e^{-r^*(s-T_0)} \frac{K}{T_C} dt \\ &= (L - myP(T_0)) + \frac{K}{r^* T_C}(1 - e^{-r^* T_C}), \end{aligned} \tag{16}$$

or, after taking logarithms and simplifying,

$$r^* = T_C^{-1} \ln\left[E_{T_0}\left(V_T\right) + \frac{K}{r^* T_C} \right] - T_C^{-1} \ln\left[(L - myP(T_0)) + \frac{K}{r^* T_C} \right]. \tag{17}$$

6. Endogenous Land Price

Under the assumption that land development is supplied by a competitive market, all rents accrue to the land since land is a scarce factor. This allows us to determine L endogenously by solving (16) for L using (15) under risk-neutrality. That is, L satisfies

$$L = \frac{K}{rT_C}(1 - e^{-rT_C}) - e^{-rT_C} \tilde{E}_{T_0}\left(V_T\right) - myP(T_0), \tag{18}$$

where

$$\tilde{E}_{T_0}\left(V_T\right) = (M - mN(d_1))P(T_0)e^{(r-\delta)T_C} + m(X(T) - yP(T_0))N(d_2). \tag{19}$$

We note that the equilibrium land price is determined by the most valuable development plan. If our developer's plan did not use the optimal K and T_C combination then other developers planning to develop optimally would offer more. Since equation (18) precludes the developer from getting any rents, it is really implicit in our model that the given K and T_C correspond to the optimal development plan, and can thus be considered as characteristics of the particular parcel of land.

7. Default

Now we consider conditions under which it is optimal for a leveraged developer to default at time T. If the developer defaults on D_T, lenders receive a proportion γ of the developer's assets in the project.

Define a random variable m^* to be the number of lots taken down by builders at time T. If builders always exercise options that are in-the-money, then

$$m^* = \begin{cases} m & \text{if} \quad P(T) > X(T) - yP(T_0) \\ 0 & \text{if} \quad P(T) \le X(T) - yP(T_0). \end{cases}$$

It is optimal for the developer to default at time T if

$$P(T) < P^*(r_d^*, m^*) \equiv \frac{D_T(r_d^*) - m^*(X(T) - yP(T_0)) - C}{M - m^*},$$

provided $M - m^* > 0$. When $M - m^* = 0$, default is optimal if and only if $D_T(r_d^*) - M(X(T) - yP(T_0)) - C > 0$.

Now define the events

$$A \equiv \{P(T) > X(T) - yP(T_0)\}$$

and

$$B \equiv \left\{P(T) \ge P^*(r_d^*, m^*)\right\}.$$

In words, A is the event that builders exercise options, and B is the event that the developer does not default. Thus $A \cap B$ is the event that options are exercised and the developer does not default. The event $A^c \cap B$ is the event that the developer does not default even though options expire out-of-the-money. The event $A \cap B^c$ is the event that options are exercised, yet the developer still defaults. Finally, $A^c \cap B^c$ is the event that options expire out-of-the-money and the developer defaults. Let P_{def} denote the minimal time T market price of a lot in order for developer default to be triggered.

If the developer defaults, the lender receives an asset with market value $\gamma(M - m^*)P(T)$ ($0 \le \gamma \le 1$). The price of insurance guaranteeing the lender's losses in the event the developer defaults is the price of a put option paying off

$$(D_T(r_d^*) - m^*(X(T) - yP(T_0)) - \gamma(M - m^*)P(T))$$

in the event that the developer defaults. When valuing this put, we must consider two cases.

Case 1. $A \cap B^c = \varnothing$, which will be the case if and only if $P^*(r_d^*, m) \le X(T) - yP(T_0)$. In this case, the developer never defaults when builders exercise options. Within Case 1, there are two possibilities that must be considered.

Case 1.a. First, if $X(T) - yP(T_0) > P^*(r_d^*, 0)$ then builder exercise always reduces P^* and consequently

$$P^*(r_d^*, m) < P^*(r_d^*, 0) < X(T) - yP(T_0)$$

So in this case, $P_{def} = P^*(r_d^*, 0)$, and the value of the put guaranteeing the lender's losses is given by

$$\begin{aligned} V^{PUT}(r_d^*) &= e^{-rT_C}\, \tilde{E}_{T_0}[D_T(r_d^*) - \gamma MP(T) \mid P(T) \le P^*(r_d^*, 0)] \\ &= e^{-rT_C} D_T(r_d^*) N(-g_2(r_d^*, 0)) - e^{-\delta T_C} \gamma MP(T_0) N(-g_1(r_d^*, 0)), \end{aligned} \tag{20}$$

where

$$g_1(r_d^*, 0) = \frac{1}{\sigma\sqrt{T_C}} \left[\ln\left(\frac{P(T_0)}{P^*(r_d^*, 0)} \right) + (r - \delta + \sigma^2/2) T_C \right], \tag{21}$$

and $g_2(r_d^*,0)=g_1(r_d^*,0)-\sigma\sqrt{T_C}$. Note that $N(-g_2(r_d^*,0))$ is the probability under the risk-neutral measure that the developer defaults.

Case 1.b. The other situation in which builder exercise and developer default can never coexist is when

$$P^*(r_d^*,m) < X(T) - yP(T_0) < P^*(r_d^*,0).$$

Here $P_{def} = X(T) - yP(T_0)$ and

$$\begin{aligned} V^{PUT}(r_d^*) &= e^{-rT_C}\tilde{E}_{T_0}[D_T(r_d^*) - \gamma MP(T) \mid P(T) \le X(T) - yP(T_0)] \\ &= e^{-rT_C} D_T(r_d^*) N(-d_2) - e^{-\delta T_C}\gamma MP(T_0) N(-d_1). \end{aligned} \tag{22}$$

Case 2. $A \cap B^c \neq \varnothing$, which will be the case if and only if $P^*(r_d^*,m) > X(T) - yP(T_0)$. In this case, developer default can occur despite the fact that builders exercise their options. This will be the case whenever

$$X(T) - yP(T_0) < P^*(r_d^*,m) < P^*(r_d^*,0)$$

or

$$X(T) - yP(T_0) < P^*(r_d^*,0) < P^*(r_d^*,m).$$

Either way $P_{def} = P^*(r_d^*,m)$. Thus the fair value of the put is

$$\begin{aligned} V^{PUT}(r_d^*) &= e^{-rT_C}\tilde{E}_{T_0}[D_T(r_d^*) - m(X(T) - yP(T_0))1_A - \gamma(M - m1_A)P(T) \mid P(T) \le P^*(r_d^*,m)] \\ &= e^{-rT_C} D_T(r_d^*) N(-g_2(r_d^*,m)) - e^{-rT_C} m(X(T) - yP(T_0))(N(d_2) - N(g_2(r_d^*,m))) \\ &\quad - e^{-\delta T_C}\gamma MP(T_0) N(-g_1(r_d^*,m)) + e^{-\delta T_C}\gamma mP(T_0)(N(d_1) - N(g_1(r_d^*,m))), \end{aligned} \tag{23}$$

where

$$g_1(r_d^*,m) = \frac{1}{\sigma\sqrt{T_C}}\left[\ln\left(\frac{P(T_0)}{P^*(r_d^*,m)}\right) + (r - \delta + \sigma^2/2)T_C\right], \tag{24}$$

and $g_2(r_d^*, m) = g_1(r_d^*, m) - \sigma\sqrt{T_C}$.

Now the equilibrium cost of debt for the developer, r_d^*, satisfies

$$e^{-r^* T_C} D_T(r_d^*) + V^{PUT}(r_d^*) = e^{-rT_C} D_T(r_d^{\,*}). \qquad (25)$$

Intuitively (25) is saying: the present value at time 0 (at the developer's cost of debt) of the maturity value of risky debt *plus* the time 0 premium for insurance against default on the risky debt *equals* the present value at time 0 of the maturity value of risky debt as if it were risk-free.

8. Expected Return on Equity With Possible Default on Debt

A. Value of the Development to the Developer

There are several distinct cases to consider corresponding to the cases in the previous section in deriving the expected terminal value of the development to the leveraged developer at time 0.

Case 1.a. If $M - m > 0$, and

$$P^*(r_d^*, m) < P^*(r_d^*, 0) < X(T) - yP(T_0),$$

then expected terminal value of the development to the leveraged developer at time T_0, $\hat{V}_T$, is

$$\begin{aligned}\hat{V}_T &= E[(M-m)P(T)1_B + m(X(T) - yP(T_0))1_{A\cap B} + mP(T)1_{A^c \cap B}] \\ &= e^{(\mu-\delta)T_C}(M-m)P(T_0)N(g_1^*(r_d^*,0)) + m(X(T) - yP(T_0))N(\min\{d_2^*, g_2^*(r_d^*,0)\}) \qquad (26) \\ &\quad + e^{(\mu-\delta)T_C} mP(T_0)\max\{N(g_1^*(r_d^*,0)) - N(d_1^*), 0\}).\end{aligned}$$

Case 1.b. If $M - m > 0$, and

$$P^*(r_d^*, m) < X(T) - yP(T_0) < P^*(r_d^*, 0).$$

then expected terminal value of the development to the leveraged developer at time T_0, $\hat{V}_T$, is

$$\begin{aligned}\hat{V}_T &= E[(M-m)P(T)1_B + m(X(T) - yP(T_0))1_{A\cap B} + mP(T)1_{A^c\cap B}] \\ &= e^{(\mu-\delta)T_C}(M-m)P(T_0)N(d_1^*) + m(X(T) - yP(T_0))N(d_2^*).\end{aligned} \quad (27)$$

Case 2. If $M - m > 0$, and

$$X(T) - yP(T_0) < P^*(r_d^*, m) < P^*(r_d^*, 0)$$

or

$$X(T) - yP(T_0) < P^*(r_d^*, 0) < P^*(r_d^*, m).$$

then expected terminal value of the development to the leveraged developer at time T_0, $\hat{V}_T$, is given by

$$\begin{aligned}\hat{V}_T &= E[(M-m)P(T)1_B + m(X(T) - yP(T_0))1_{A\cap B} + mP(T)1_{A^c\cap B}] \\ &= e^{(\mu-\delta)T_C}(M-m)P(T_0)N(g_1^*(r_d^*, m)) + m(X(T) - yP(T_0))N(\min\{d_2^*, g_2^*(r_d^*, m)\}) \quad (28) \\ &\quad + e^{(\mu-\delta)T_C}mP(T_0)\max\{N(g_1^*(r_d^*, m)) - N(d_1^*), 0\}).\end{aligned}$$

Case 3. If $M - m = 0$ and $D_T(r_d^*) - M(X(T) - yP(T_0)) \le 0$, then expected terminal value of the development to the leveraged developer at time T_0 is

$$\begin{aligned}\hat{V}_T &= E[M(X(T) - yP(T_0))1_A + MP(T)1_{A^c} \mid F(T_0)] \\ &= M(X(T) - yP(T_0))N(d_2^*) + e^{(\mu-\delta)T_C}MP(T_0)N(-d_1^*).\end{aligned} \quad (29)$$

Case 4. If $M - m = 0$ and $D_T(r_d^*) - M(X(T) - yP(T_0)) > 0$, then expected terminal value of the development to the leveraged developer at time T_0 is

$$\hat{V}_T = 0 \tag{30}$$

2. Expected Return on Equity

Now let $\hat{r}_e$ denote the developer's expected return on equity from the project when default is possible. Then

$$\begin{aligned}(1-\alpha_1)(L - myP(T_0)) + (1-\alpha_2)\frac{K}{\hat{r}_e T_C}(1 - e^{-\hat{r}_e T_C}) \\ = e^{-\hat{r}_e T_C}\{\hat{V}_T - \alpha_1 e^{r_d T_C}(L - myP(T_0))N(h^*) - \alpha_2 \frac{K}{r_d T_C}(e^{r_d T_C} - 1)N(h^*)\}.\end{aligned} \tag{31}$$

where

$$h^* = \begin{cases} g_2^*(r_d^*, 0) & \text{if} \quad P_{def} = P^*(r_d^*, 0) \\ d_2^* & \text{if} \quad P_{def} = X(T) - yP(T_0) \\ g_2^*(r_d^*, m) & \text{if} \quad P_{def} = P^*(r_d^*, m). \end{cases}$$

So $\hat{r}_e$ satisfies

$$e^{\hat{r}_e T_C} = \frac{\hat{V}_T - \alpha_1 e^{r_d T_C}(L - myP(T_0))N(h^*) - \alpha_2 \frac{K}{r_d T_C}(e^{r_d T_C} - 1)N(h^*) + (1-\alpha_2)\frac{K}{\hat{r}_e T_C}}{(1-\alpha_1)(L - myP(T_0)) + (1-\alpha_2)\frac{K}{\hat{r}_e T_C}} \tag{32}$$

or

$$\begin{aligned}\hat{r}_e = T_C^{-1}\ln\left[V_T - \alpha_1 e^{r_d T_C}(L - myP(T_0))N(h^*) - \alpha_2 \frac{K}{r_d T_C}(e^{r_d T_C} - 1)N(h^*) + (1-\alpha_2)\frac{K}{\hat{r}_e T_C}\right] \\ -T_C^{-1}\ln\left[(1-\alpha_1)(L - myP(T_0)) + (1-\alpha_2)\frac{K}{\hat{r}_e T_C}\right]\end{aligned} \tag{33}$$

The next section provides the results from the numerical solution of the model described above.

III. Numerical Solutions and Analysis

3.1 Base Case Parameter Values

Table 1 presents the base case parameters from which we can derive and compare results. We normalize the completed lot price at time zero, P_0, to be 100. We set the drift term of the lot price, μ, to be 8%, and the convenience yield, δ, is set to be 1%. The drift rate represents a 3% premium over the risk free rate, which is set to 5%, to account for the relatively low market risk inherent in residential land ownership.[6] The convenience yield on the lot price reflects the value of keeping completed lots in inventory less any ownership costs, such as property taxes, for holding the property. The volatility of the process, σ, is set to 12.5% which is similar to the volatility on built properties.

[6] Land or lot prices, especially those entitled for residential construction, are not highly correlated with market returns, so a lower risk premium relative to other commercial real estate is justified. According to surveys and historical data, commercial built property has a risk premium of approximately 4-5%.

Table 1:
Base Case Model Parameters.

Parameter	Base Case Value
P(t) spot price of the completed lot.	$P_0 = 100$
μ is the drift of the completed price over time	$\mu = .08$
δ is the cash flow yield form the completed lot	$\delta = 01$
σ is the constant volatility rate	$\sigma = .125$
K is the construction cost to complete the lot for sale to the builder	K = 80
T_c is the time to construction completion	$T_c=1$
X is the exercise price of the builder's option to purchase the completed lot	$X = Pe^{r_d T} = 105.13$
M is the total number of lots	Normalized to M=1
m is the percentage of lots presold	m=0, 25%, 50%, 75%, 100%
α_1 is the percentage of the initial cash flow financed, i.e., the land purchase price less any option binder deposits received	$\alpha_1 = 80\%$
α_2 is the percentage of construction costs financed	$\alpha_2 = 80\%$
γ is the percentage of lot value retained by lender upon developer default	$\gamma = 70\%$
r_d the risk-free rate	$r_d = 5\%$

Construction costs are set to be equal to 80% of the total completed lot value or $80. This is chosen such that raw land values represent approximately 20-25% of the completed lot value which is common in the residential industry. Because the problem is homogenous of degree one in M, the number of lots, and m the number of lots presold, we normalize M to be equal to 1. We vary the number of presales, to vary from 0% to 100%.

Debt financing is set to be α_1 = 80% of the initial cash out of pocket, i.e., the land purchase costs less any proceeds from the sale of presale options, and α_2 = 80% for the financing of the construction costs,. Upon default the lender is assumed to take back the property and receive γ = 70% of its current value. Finally, the presale option exercise price is equal to the initial lot price increased by the risk free rate over the time of construction, which is set to one year.

Table 2 displays results for varying levels of volatility. In Table 2a for the base case, σ = 12.5%, the levered (unlevered) expected rates of return vary from 22.74% (9.90%)

with no presales to 9.62% (6.04%) for the case where all lots are presold. The reduction in expected returns declines with the level of presales to reflect the shifting of risk from the developer to the lot purchaser (referred to as the builder). The levered returns without presales are similar to those reported in return surveys of developers. What is interesting is that these high returns are only justified when there is no risk management involved, i.e., no presales for the project. Note that levered returns decline for all levels of presales as volatility is increased. This is due to the lender taking more of the return to compensate them for the additional risk on the debt as volatility in property prices increases.[7]

Table 2b show the endogenously determined spread on the debt financing over the risk free rate for varying levels of presales. For the base case, spreads on debt are 171 basis points with no presales and only 15 basis points when all lots are presold. Presales decrease spreads for tow reasons. First, presales reduce the probability of developer default on the debt and therefore lower expected default costs. Table 2c shows that the probability of default increases with property price volatility but is lowered with additional presales. Second, with more presales, less upfront financing is needed as the option premium from the presales is essentially used as an equity investment into the property.

[7] Developers would maximize property value by taking in no debt and avoiding default costs. However we assume that capital constraints exist in the private equity market and that debt financing is required.

Table 2a:
Expected Return on Unlevered and Levered Equity
for Varying Values of Price Volatility σ

m, percentage of lots presold	σ = .10		Base Case σ = .125		σ = .15	
	Unlevered	Levered	Unlevered	Levered	Unlevered	Levered
0%	9.90%	26.21%	9.90%	22.74%	9.90%	17.59%
25%	9.06%	23.17%	9.08%	21.21%	9.11%	18.31%
50%	8.14%	19.46%	8.17%	18.40%	8.21%	16.81%
75%	7.15%	15.13%	7.16%	14.52%	7.20%	13.64%
100%	6.07%	10.14%	6.04%	9.62%	6.03%	8.98%

Table 2b:
Spread on Construction Financing in basis points
for Varying Values of Price Volatility σ

m, percentage of lots presold	σ = .10	Base Case σ = .125	σ = .15
	Spread in bps on Debt	Spread in bps on Debt	Spread in bps on Debt
0%	51	171	364
25%	29	98	209
50%	16	54	117
75%	8	29	63
100%	4	15	32

Table 2c:
Probability of Builder Exercise and Probability of Developer Default
for Varying Values of Price Volatility σ

m, percentage of lots presold	σ = .10	Base Case σ = .125	σ = .15
	Probability of Builder Exercise = 82.65% in All Cases		
	Probability of Developer Default	Probability of Developer Default	Probability of Developer Default
0%	0.71%	3.01%	7.16%
25%	0.38%	1.65%	3.98%
50%	0.20%	0.88%	2.17%
75%	0.10%	0.45%	1.13%
100%	0.04%	0.22%	0.55%

In Table 3 we show results for varied levels of debt financing. As debt financing is increased to 85%, the spread on debt increases dramatically to reflect an increasing probability of default (it more than doubles from the base case) and the expected costs of default. As with increasing volatility, the increasing spread on debt depresses expected levered equity returns as default costs eat up the return to equity. Lowering leverage from 80% to 75% lowers the cost of debt dramatically and maintains expected levered returns.

This illustrates the likely high cost off raising private equity capital since high levels of debt are used in the development industry despite it cost.

Table 3a:
Expected Return on Levered Equity
for Varying Values of Debt Financing $\alpha_1 = \alpha_2$

m, percentage of lots presold	$\alpha_1 = \alpha_2 = .75$	$\alpha_1 = \alpha_2 = .80$	$\alpha_1 = \alpha_2 = .85$
	Expected Levered Equity Return	Expected Levered Equity Return	Expected Levered Equity Return
0%	22.54%	22.74%	8.79%
25%	20.02%	21.21%	15.79%
50%	16.94%	18.40%	16.51%
75%	13.30%	14.52%	13.94%
100%	9.04%	9.62%	9.01%

Table 3b:
Spread on Construction Financing in basis points
for Varying Values of Debt Financing $\alpha_1 = \alpha_2$

m, percentage of lots presold	$\alpha_1 = \alpha_2 = .75$	$\alpha_1 = \alpha_2 = .80$	$\alpha_1 = \alpha_2 = .85$
	Spread in Basis Points on Debt	Spread in Basis Points on Debt	Spread in Basis Points on Debt
0%	44	171	625
25%	24	98	340
50%	12	54	192
75%	6	29	107
100%	3	15	58

Table 3c:
Probability of Builder Exercise and Probability of Developer Default
for Varying Values of Debt Financing $\alpha_1 = \alpha_2$

m, percentage of lots presold	$\alpha_1 = \alpha_2 = .75$	$\alpha_1 = \alpha_2 = .80$	$\alpha_1 = \alpha_2 = .85$
	Probability of Builder Exercise = 82.65% in All Cases		
	Probability of Developer Default	Probability of Developer Default	Probability of Developer Default
0%	0.70%	3.01%	12.07%
25%	0.36%	1.65%	6.30%
50%	0.18%	0.88%	3.41%
75%	0.08%	0.45%	1.82%
100%	0.04%	0.22%	0.94%

In table 4 we vary the time to completion for the project. Increasing time to complete the project increases the cumulative variance of completed lot values, which in turn increase the probability of a low lot price and eventual developer default. Note however that as exercise price of the presale option is held constant, that the probability of builder exercise of the presale option increases as time and the positive drift of completed property prices overcomes the increase in cumulative volatility. This is illustrated in Tables 4b and 4c in the increasing financing spread and probability of developer default.

Table 4a:
Expected Return on Levered Equity
for Varying Values of Time to Completion T

m, percentage of lots presold	T=0.5	Base Case T=1	T=2
	Expected Levered Equity Return	Expected Levered Equity Return	Expected Levered Equity Return
0%	27.38%	22.74%	18.04%
25%	24.75%	21.21%	20.59%
50%	21.88%	18.40%	19.39%
75%	18.78%	14.52%	14.60%
100%	15.47%	9.62%	5.38%

Table 4b:
Spread on Construction Financing in Basis Points
for Varying Values of Time to Completion T

m, percentage of lots presold	T=0.5	Base Case T=1	T=2
	Spread in Basis Points on Debt	Spread in Basis Points on Debt	Spread in Basis Points on Debt
0%	44	171	319
25%	31	98	117
50%	22	54	28
75%	16	29	6
100%	11	15	1

Table 4c:
Probability of Builder Exercise and Probability of Developer Default
for Varying Values Time to Completion T

m, percentage of lots presold	T=0.5	Base Case T=1	T=2
	Probability of Builder Exercise 58.39%	Probability of Builder Exercise 82.65%	Probability of Builder Exercise 99.21%
	Probability of Developer Default	Probability of Developer Default	Probability of Developer Default
0%	0.42%	3.01%	9.87%
25%	0.30%	1.65%	4.05%
50%	0.21%	0.88%	0.66%
75%	0.14%	0.45%	0.11%
100%	0.10%	0.22%	0.01%

In Table 5, we vary the drift rate of the completed property price over time. The drift rate plus the convenience yield reflects the risk adjusted required unleverd return on the equity. Thus, returns on levered equity increase as the risk adjusted unlevered returns (and unlevered risk premiums) increase. In today's low risk premium environment, it is likley that return on levered equity are closer to the 15-20% range for development projects without presales as is shown in table 5a for the μ=.07 and the base case μ=.08 results.

Because risk adjusted returns don't affect debt pricing, spreads remain the same as in the base case and are therefore not shown in Table 5. This is because risk-neutral pricing results are not affected by risk adjusted return parameters. However table 5b shows that *real* probabilities of builder default decrease with the drift rate as the probabilities of very low property prices is mitigated. *Real* builder exercise probabilities also increase with the risk adjusted rate.

Table 5a:
Expected Return on Levered Equity
for Varying Values of Expected Growth Rate in Price μ

m, percentage of lots presold	μ=.07	Base Case μ=.08	μ=.09
	Expected Levered Equity Return	Expected Levered Equity Return	Expected Levered Equity Return
0%	15.40%	22.74%	29.78%
25%	15.06%	21.21%	27.12%
50%	13.58%	18.40%	23.04%
75%	11.23%	14.52%	17.65%
100%	8.12%	9.62%	10.93%

Table 5b:
Probability of Builder Exercise and Probability of Developer Default
for Varying Values Expected Growth Rate in Price μ

m, percentage of lots presold	μ=.07	Base Case μ=.08	μ=.09
	Probability of Builder Exercise 80.53%	Probability of Builder Exercise 82.65%	Probability of Builder Exercise 84.63%
	Probability of Developer Default	Probability of Developer Default	Probability of Developer Default
0%	3.60%	3.01%	2.50%
25%	2.01%	1.65%	1.35%
50%	1.09%	0.88%	0.71%
75%	0.56%	0.45%	0.35%
100%	0.28%	0.22%	0.17%

In Table 6 we vary the convenience yield. The convenience yield reflects the value of keeping completed lots in inventory for both the developer and the builder less any ownership costs, such as property taxes, for holding the property. As convenience yield increases, less of the property's risk adjusted return comes from appreciation on the completed property price. Therefore, the probability of default by the builder increases slightly over the range of convenience yields that we have chosen to compare. This is reflected in slightly increased spreads on the debt financing.

Table 6a:
Expected Return on Levered Equity
for Varying Values of Convenience Yield δ

m, percentage of lots presold	δ =0	Base Case δ =.01	δ =.02
	Expected Levered Equity Return	Expected Levered Equity Return	Expected Levered Equity Return
0%	22.59%	22.74%	22.90%
25%	21.42%	21.21%	21.24%
50%	18.33%	18.40%	18.68%
75%	13.72%	14.52%	15.33%
100%	7.67%	9.62%	11.24%

Table 6b:
Spread on Construction Financing in Basis Points
for Varying Values of Convenience Yield δ

m, percentage of lots presold	δ =0	Base Case δ =.01	δ =.02
	Spread in Basis Points on Debt	Spread in Basis Points on Debt	Spread in Basis Points on Debt
0%	171	171	171
25%	80	98	110
50%	36	54	70
75%	14	29	43
100%	5	15	26

Table 6c:
Probability of Builder Exercise and Probability of Developer Default
for Varying Values of Convenience Yield δ

m, percentage of lots presold	δ =0	Base Case δ =.01	δ =.02
	Probability of Builder Exercise 91.04%	Probability of Builder Exercise 82.65%	Probability of Builder Exercise 74.79%
	Probability of Developer Default	Probability of Developer Default	Probability of Developer Default
0%	3.02%	3.01%	3.01%
25%	1.34%	1.65%	1.88%
50%	0.56%	0.88%	1.15%
75%	0.21%	0.45%	0.69%
100%	0.07%	0.22%	0.40%

IV. Conclusion

In this paper we have developed a contingent claims model of the development process. This model is rich enough to examine virtually every aspect of development. Perhaps most importantly, the model generates an endogenous land value based upon the development opportunities the developer faces. The model explicitly incorporates the risk to the developer of builder default. Returns results from the model are consistent with the high rates of return observed in the empirical studies.

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