

Pricing Property Index Linked Swaps with Counterparty Default Risk

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March 2006

Ricardo Pereira thanks the financial support from Fundação para a Ciência e Tecnologia.

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Kanak Patel* and Ricardo Pereira⁺

Abstract

This paper extends Bjork and Clapham (2002) model for pricing real estate index total return swaps. Our extension considers counterparty default risk within a first passage contingent claims model. We price total return swaps on property indices with different levels of default risk. We develop this model under same assumptions as Bjork and Clapham (2002) and find that total return swap price is no longer zero. Total return swap payer must charge a spread over the market interest rate that compensates him for the exposure to this additional risk. Based on commercial property indices in the UK, we observe that the computed spreads are much lower than a sample of quotes obtained from one of the traders in the market.

JEL: C30; G13; G33

Keywords: total return swaps, property derivatives, default risk

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1. Introduction

Risk sharing in property markets is a vexing issue not only because property is the biggest store of wealth in the society but also because property markets are prone to speculative whips of investors as equity markets. Property owners in the UK who have witnessed soaring property values in recent years are no doubt worried that they cannot protect the values of their assets without having to sell them. Likewise, institutional investors and portfolio managers who could not gain an exposure to property markets without directly or indirectly owning the assets must have been equally frustrated. Greater risk sharing and an easier entry/exit access in these markets is a major concern to investors, portfolio managers, and banks that hold large collateralised loan portfolios on their balance sheets. Property derivatives can allow investors to increase/reduce exposure to property market, hedge current position, as well as change portfolio composition without having to buy or sell the physical asset.

For banks, property assets figure mostly off-balance sheet as collateral for loans. The purpose of the collateral is to limit downside losses if the borrower defaults on the loan. Consequently, banks are exposed to property market risk. While several types of credit derivatives (such as credit default swaps) allow banks to hedge their credit risk, the market is not yet sufficiently wide enough to allow them to hedge credit risk of their entire portfolio of claims.

In the early 1990s, the London Futures and Options Exchange (FOX) launched four index based futures contracts in order to facilitate greater risk sharing and accessibility to residential and commercial property markets in the UK. These contracts, however,

failed to gain sufficient liquidity and were quickly withdrawn. Since then there has been a steady activity in the over-the-counter property income certificates, a type of bond with an embedded return based on Investment Property Databank (IPD) index, sold by Barclays Capital and Protego. The two main types of property derivatives currently available in the market are property index certificates and total rate of return real estate index swaps (TRS). ABN AMRO bank led the recent attempt in the UK to provide two-way prices on property derivatives for all property and sector based swaps linked to total return on Investment Property Databank's All Property Index (Euroweek, 2005). Eurohypo, Deutsche Bank, Tullett Prebon and ICAP are also seeking to provide such deals in the market. A TRS is a bilateral contract between a total return payer, who owns the asset, and a total return receiver, who will enjoy the asset's cash flows or returns without owning it. The total return payer pays the total return of the underlying asset and receives, from the total return receiver, a floating or fixed rate payment (when floating, this payment can be linked to LIBOR). At the TRS's maturity date, the receiver must pay to the payer any depreciation in the value of the underlying asset value as shown in Figure 1. One particular feature of this off-balance sheet contract is that, like a debt contract, it can give either positive or negative value to the counterparty at any given time.

Considering that TRS payer is a bank, the bank transfers property assets' market risk or volatility to the TRS receiver. Assuming that IPD is the right proxy for a bank's property asset portfolio (on and off balance sheet), this transaction allows the bank to hedge some of its real estate assets' market risk and consequently reduce the economic capital requirement. Basel Committee on Banking Supervision (BCBS, 2004), under Basel II Accord Standardized approach, recommends that claims secured by residential

or commercial property should be risk weighted at 35 and 100 percent, respectively. This risk weighting approach for secured loans is, however, justifiable only for assets that lack market liquidity. In a world where banks can swap real estate assets' market risk, using a TRS, these risk weightings must be revised. The impact of TRS on capital requirements is a crucial factor in the development of property derivatives market.

The literature on property derivatives valuation is scant. Nonetheless, given TRS cash flows, we can value swaps by combining two distinct derivative valuation models: credit default swap and interest rate swap valuation models. Houweling and Vorst (2005) provide good literature review of credit default swaps valuation models. The latter valuation models are described in detail in Sun, Sundaesan and Wang (1993) and Minton (1997). To the best of our knowledge, only two papers describe real estate linked TRS specific valuation models. Buttimer, Kau and Slawson (1997) develop a two state model for pricing a TRS dependent on a real estate index as well as an interest rate. The authors use a bivariate binomial model to value a commercial real estate indexed linked swap. Assuming that index value follows a geometric Brownian motion and the interest rate CIR model, swap's value is positive, although near zero. Bjork and Clapham (2002) point out several limitations of this model and demonstrate that value of the swap should be zero. They develop an arbitrage-free framework that is more general than Buttimer et al (1997). TRS value equals the sum of individual swaplets, which are active over a small time interval. Although the latter model is theoretically robust and sound in an arbitrage-free world, it does not allow for the existence of counterparty default risk. In our opinion, TRS fair value is no longer zero if we take into account counterparty default risk because in the short term TRS value can be positive or negative. Thus it is essential to incorporate counterparty default risk in TRS pricing

model. Under this scenario, in an arbitrage-free world, TRS payer must charge a spread over the reference interest rate. This spread should take into account special features of underlying property assets, such as lower liquidity and transparency in the market. This spread can be particularly important in property derivative products because, given infrequent trading in property markets, swap traders have to rely on valuations rather than actual market prices of underlying properties for pricing counterparty default risk in these markets.

The purpose of this paper is to extend Bjork and Clapham (2002) pricing model for a TRS payer assuming counterparty default risk. Cooper and Mello (1991) also discuss the problem of counterparty default risk, but they follow a different line of investigation by first pricing each counterparty promised gross payment separately and then adding those values. Duffie and Huang (1996) show that this procedure is erroneous and leads to an overestimation of default risk in a swap. Our results show that the spread over market interest rate that TRS payer must charge is highly dependent on the volatility of index returns and on counterparty default risk. The higher the volatility of returns, and counterparty default risk, the higher the spread over market interest rate. Based on quotes from one of the traders of this type of property derivatives, we observe that computed spreads underestimate the spreads quoted by traders in the market. An over-the-counter TRS on a US real estate index, which closely resembles our TRS contract, was also traded with a spread consistent with our results. This underestimation apparently arises because the market spread incorporates other components besides counterparty default risk, which are not considered in our analysis. In practice, the trader assumes additional exposures when he trades property derivatives. To evaluate

the performance and accuracy of the model more precisely, it is necessary to quantify these additional components.

The rest of the paper is organised as follows: Section 2 presents and develops a model to value TRS with counterparty default risk. Section 3 applies the model to several real estate indices, with different maturities and different counterparty default probabilities. This section also presents some evidence using market data on similar TRS. Section 4 concludes the paper.

2. Valuation of Total Return Swaps with Counterparty Default Risk

In this section, we follow Buttner et al (1997) and Bjork and Clapham (2002) procedures for valuation of TRS swaps with counterparty default risk. Bjork and Clapham (2002) argue that a total return property index linked swap can be valued as a sum of individual swaplets, which are active over a period of time denoted by $\Delta = t_k - t_{k-1}$. At the end of each time period, the TRS payer will have the following cash flows (TRS receiver will have the opposite cash flows) from the active swaplet:

- receive an amount equal to the spot market rate at t_{k-1} , such as LIBOR, plus a spread, δ , that for now we assume to be equal to zero, for the period Δ , times the notional amount and any depreciation of the index, I_t , over the time period;
- pay the total return of the index (appreciation plus income) generated over the time period.

Consider now the following trading strategy, that starts at time t and ends at time t_n , and which is repeated at each time period $[t_{k-1}, t_k]$, for $k = 1, \dots, n$.

- At time t_{k-1} , sell the index I_{k-1} and lend this amount over the time period Δ at the spot LIBOR $L = L(t_{k-1}, t_k)$.
- At t_k buy the index by I_t . Receive the principal of the loan, I_{k-1} , plus the accrued interest, ΔLI_{k-1} .

This trading strategy exactly replicates the cash flows of TRS payer. Since this strategy is self-financing and the initial cost of setting it is zero, its arbitrage free value must equal zero. This is also the value of TRS.

Under the above considerations, regardless of the assumptions about the interest rate and the index value stochastic process, Bjork and Clapham (2002) show that the arbitrage-free price of the TRS is zero.

Assumptions

Uncertainty in the economy is modelled by a filtered probability space (Ω, \mathcal{F}, P) , where Ω represents the set of possible states of nature, \mathcal{F}_t is the information available to investors over time t and P is the probability measure. Besides this, we assume that the index process, I_t , is ex dividend and that there is a cumulative dividend (income) process D_t . The holder of the index receives over the interval $(s, t]$ the amount $D_t - D_s$. The risk-free numeraire (or money market account) value, at time t , B_t , follows the process: $e(\int_0^t r_t dt)$, where r_t denotes the short-term interest rate, which can be deterministic or modelled by a stochastic process. Bond market is liquid and there are bonds of all possible maturities. The price at time t of a zero coupon bond that matures

at T is denoted by $p(t, T)$. We assume a perfect arbitrage-free market, where there exists an equivalent martingale measure $Q \sim P$. Therefore:

- The normalized gains process G_t^B in a risk neutral world is

$$G_t^B = \frac{I_t}{B_t} + \int_0^t \frac{1}{B_s} dD_s \quad (1)$$

- Bond prices are defined by

$$p(t, T) = E^Q \left[e^{-\int_t^T r_s ds} \mid \mathcal{F}_t \right] \quad (2)$$

- In a risk neutral world, the arbitrage free price process $\Pi(t; X)$ at time t of a contingent claim X , paid out at time T is expressed by

$$\Pi(t; X) = E^Q \left[e^{-\int_t^T r_s ds} X \mid \mathcal{F}_t \right] \quad (3)$$

Under these assumptions, assuming no counterparty default risk, the arbitrage-free value of a TRS at time t , considering it as a sum of swaptlets, is

$$\Pi(t; \text{TRS}) = \sum_{k=1}^n \Pi(t; X_k)$$

X_k represents the TRS payer net payments, at time t_k ,

$$X_k = (I_k - I_{k-1}) + \int_{t_{k-1}}^{t_k} e^{\int_s^{t_k} r_u du} dD_s - \Delta L(t_{k-1}, t_k) I_{k-1}$$

where the first term equals index appreciation, the second term represents the value, at time t_k , of the index dividend, produced during Δ , and the third term equals the cash inflow. In a risk neutral world, using (3), we have

$$\begin{aligned} \Pi(t; X_k) &= E^Q \left[e^{-\int_t^{t_k} r_s ds} X_k \mid \mathcal{F}_t \right] \\ &= E^Q \left[e^{-\int_t^{t_k} r_s ds} I_k \mid \mathcal{F}_t \right] - E^Q \left[e^{-\int_t^{t_k} r_s ds} I_{k-1} \mid \mathcal{F}_t \right] + \end{aligned}$$

$$+ E^Q \left[e^{-\int_t^{t_k} r_s ds} \int_{t_{k-1}}^{t_k} e^{\int_s^{t_k} r_u du} dD_s \mid \mathcal{F}_t \right] - E^Q \left[e^{-\int_t^{t_k} r_s ds} \Delta L(t_{k-1}, t_k) I_{k-1} \mid \mathcal{F}_t \right]$$

After simplification (see Bjork and Clapham (2002), we have:

$$\Pi(t; X_k) = B_t E^Q [G_k^B - G_{k-1}^B \mid \mathcal{F}_t] \quad (4)$$

Since the normalized gains process is a martingale under Q , the arbitrage free value of the swap must be zero. However, when taking into account counterparty default risk in the valuation, the fair spread over the market rate is no longer zero. Within this framework, the TRS payer's net payments, at time t_k , is

$$X_k = (I_k - I_{k-1}) + \int_{t_{k-1}}^{t_k} e^{\int_t^s r_u du} dD_s - \Delta [L(t_{k-1}, t_k) + \delta] I_{k-1}$$

where δ is the spread over the market rate.

$$\begin{aligned} \Pi(t; X_k) &= E^Q \left[e^{-\int_t^{t_k} r_s ds} X_k \mid \mathcal{F}_t \right] \quad (5) \\ &= E^Q \left[e^{-\int_t^{t_k} r_s ds} I_k \mid \mathcal{F}_t \right] - E^Q \left[e^{-\int_t^{t_k} r_s ds} I_{k-1} \mid \mathcal{F}_t \right] + \\ &+ E^Q \left[e^{-\int_t^{t_k} r_s ds} \int_{t_{k-1}}^{t_k} e^{\int_s^{t_k} r_u du} dD_s \mid \mathcal{F}_t \right] - E^Q \left[e^{-\int_t^{t_k} r_s ds} \Delta [L(t_{k-1}, t_k) + \delta] I_{k-1} \mid \mathcal{F}_t \right] \\ &= E^Q \left[e^{-\int_t^{t_k} r_s ds} I_k \mid \mathcal{F}_t \right] - E^Q \left[e^{-\int_t^{t_k} r_s ds} I_{k-1} \{1 + \Delta [L(t_{k-1}, t_k) + \delta]\} \mid \mathcal{F}_t \right] + \\ &+ E^Q \left[e^{-\int_t^{t_k} r_s ds} \int_{t_{k-1}}^{t_k} e^{\int_s^{t_k} r_u du} dD_s \mid \mathcal{F}_t \right] \end{aligned}$$

The second term can be written as

$$E^Q \left[e^{-\int_t^{t_k} r_s ds} I_{k-1} \{1 + \Delta [L(t_{k-1}, t_k) + \delta]\} \mid \mathcal{F}_t \right]$$

$$\begin{aligned}
&= E^Q \{ E^Q [e^{-\int_t^{t_k} r_s ds} I_{k-1} \{ 1 + \Delta [L(t_{k-1}, t_k) + \delta] \} | \mathcal{F}_{t_{k-1}}] | \mathcal{F}_t \} \\
&= E^Q \{ E^Q [e^{-\int_t^{t_{k-1}} r_s ds} I_{k-1} \{ 1 + \Delta [L(t_{k-1}, t_k) + \delta] \}] E^Q [e^{-\int_{t_{k-1}}^{t_k} r_s ds} | \mathcal{F}_{t_{k-1}}] | \mathcal{F}_t \} \\
&= E^Q \{ e^{-\int_t^{t_{k-1}} r_s ds} I_{k-1} \{ 1 + \Delta [L(t_{k-1}, t_k) + \delta] \} \frac{1}{1 + \Delta L(t_{k-1}, t_k)} | \mathcal{F}_t \} \\
&= E^Q \{ e^{-\int_t^{t_{k-1}} r_s ds} I_{k-1} [1 + \frac{\Delta \delta}{1 + \Delta L(t_{k-1}, t_k)}] | \mathcal{F}_t \}
\end{aligned}$$

Substituting into Equation (5), we obtain

$$\begin{aligned}
\Pi(t; X_k) &= E^Q [e^{-\int_t^{t_k} r_s ds} I_k | \mathcal{F}_t] - E^Q \{ e^{-\int_t^{t_{k-1}} r_s ds} I_{k-1} [1 + \frac{\Delta \delta}{1 + \Delta L(t_{k-1}, t_k)}] | \mathcal{F}_t \} + \\
&\quad + E^Q [e^{-\int_t^{t_k} r_s ds} \int_{t_{k-1}}^{t_k} e^{\int_s^{t_k} r_u du} dD_s | \mathcal{F}_t] \\
&= B_t E^Q [\frac{I_k}{B_k} - \frac{I_{k-1}}{B_{k-1}} (1 + \frac{\Delta \delta}{1 + \Delta L(t_{k-1}, t_k)}) + \int_{t_{k-1}}^{t_k} \frac{1}{B_s} dD_s | \mathcal{F}_t] \\
&= B_t E^Q [G_k^B - G_{k-1}^B - \frac{I_{k-1}}{B_{k-1}} \frac{\Delta \delta}{1 + \Delta L(t_{k-1}, t_k)} | \mathcal{F}_t]
\end{aligned}$$

The last term inside brackets denotes the present value of the amount $(I_{k-1} \Delta \delta)$ paid at time t_k . The arbitrage-free value of this amount must be equal to the expected loss incurred by TRS payer given TRS receiver default risk. The counterparty default risk arises only when TRS value is negative for receiver; otherwise TRS receiver has a net inflow. When TRS value is negative and TRS receiver defaults, we assume that TRS payer receives a proportion $[1 - \text{Loss Given Default (LGD)}]$ of TRS value. The loss to TRS payer, given counterparty default, is therefore, similar to the payoff of a contingent claim that pays $LGD[\max(X_k, 0)]$ at time t_k . This contingent claim can be seen as a

European call option on X_k with exercise price zero and maturity date t_k or a European call option on the underlying index value, with spot price I_{k-1} and strike price $I_{k-1}\Delta L(t_{k-1}, t_k)$ and maturity date t_k . X_k represents TRS value without counterparty default risk. Thus, present value of the amount $(I_{k-1} \Delta \delta)$ paid at time t_k must equals

$$\begin{aligned}
E^Q \left[\frac{I_{k-1}}{B_{k-1}} \frac{\Delta \delta}{1 + \Delta L(t_{k-1}, t_k)} \mid \mathcal{F}_t \right] &= E^Q \left[\text{LGD} e^{-\int_t^{t_k} r_s ds} [\max(X_k, 0)] f(t_{k-1}, t_k) \mid \mathcal{F}_t \right] \quad (6) \\
&= E^Q \left[\text{LGD} e^{-\int_t^{t_{k-1}} r_s ds} c(X_k) f(t_{k-1}, t_k) \mid \mathcal{F}_t \right] \\
&= E^Q \left[\text{LGD} \frac{1}{B_{k-1}} c(X_k) f(t_{k-1}, t_k) \mid \mathcal{F}_t \right]
\end{aligned}$$

where $f(t_{k-1}, t_k)$ represents the probability of default by the counterparty between times t_{k-1} and t_k as seen at time t and $c(X_k)$ denotes the value at time t_{k-1} of a call option on X_k with exercise price zero and maturity date t_k . Expression (6) can be simplified to write the spread as

$$\delta = \frac{(\text{LGD}c(X_k)f(t_{k-1}, t_k))(1 + \Delta L(t_{k-1}, t_k))}{\Delta I_{k-1}} \quad (7)$$

If probability of default by the counterparty or LGD equals zero there is no counterparty default risk and the spread should be zero.

To empirically apply this model we need the following additional assumptions.

- The value of the index, I , and the value of counterparty assets, V , are independent and follows a Geometric Brownian motion

$$dI = \mu_I I dt + \sigma_I I dz_I$$

$$dV = \mu_V V dt + \sigma_V V dz_V$$

where μ_I and μ_V are the risk neutral expected growth rate of the index value and counterparty assets value, respectively. For a non-dividend paying asset, $r = \mu$ is the risk free rate and for a dividend paying asset $\mu = r - \theta$, where θ denotes dividend rate. σ_I and σ_V are the volatility of the index value and counterparty assets value. z_I and z_V are variables that follow a Wiener process.

- LGD is assumed to be 49 percent (see, for example, Longstaff and Schwartz (1995), Eom et al. (2004)).

- Counterparty defaults when the value of the asset fall below a specified level, K , the threshold level, which may change over time. Several studies present closed-form solutions to compute this default probability (see, for example, Black and Cox (1976), Ericsson and Reneby (1998) and Bielecki and Rutkowski (2001)). We use the formula presented in Black and Cox (1976)¹. According to Black and Cox (1976), with some simplifications, the risk-neutral probability of default before time τ , where τ is the time of default, given by $\tau = \inf \{s \geq t \mid V_s \leq K\}$, is

$$P[\tau \leq T \mid \mathcal{F}_t] = 1 - N\left(\frac{\ln(V_t / K) + (r - \theta - 0.5\sigma_V^2)(\tau - t)}{\sqrt{\sigma_V^2(\tau - t)}}\right) + \left(\frac{V_t}{K}\right)^{1-2(r-\theta)/\sigma_V^2} N\left(\frac{\ln(K / V_t) + (r - \theta - 0.5\sigma_V^2)(\tau - t)}{\sqrt{\sigma_V^2(\tau - t)}}\right) \quad (8)$$

Equation (8) allows us to compute counterparty default probability over TRS life. Instead of using (8), we can use company's credit rating if counterparty has credit rating given by a credit rating company. These default probabilities here are estimated from

¹ Different closed-form formulae provide almost indistinguishable default probabilities. Therefore, the results are not sensitive to this parameter calculation.

credit rating of company's database or watchlist. Since, over the life of TRS there is, normally, more than one payment that occurs at the end of Δ , we need to compute counterparty default probabilities over each Δ given \mathcal{F}_t or as seen at time t . Hull (1989) defines the probability of default during the time interval Δ as

$$P[t_{k-1} \leq \tau \leq t_k | \mathcal{F}_t] = \exp(-ht_{k-1}) - \exp(-ht_k) \quad (9)$$

where h is the hazard rate or default intensity. A typical assumption is that the hazard rate, h , is constant over the period. Li (2000) shows that counterparty survival time follows an exponential distribution with parameter h . Under this assumption, the default probability over the time interval $[t, t + x]$, for $0 < x < 1$, equals one minus survival probability

$$P[\tau \leq t + x | \mathcal{F}_t] = 1 - \exp(-hx) \quad (10)$$

The additional assumption for the value of the index process will allow us to use the standard Black and Scholes (1973) model to compute European call option value.

As mentioned earlier, the spread incorporates other components besides counterparty default risk. Therefore, if these other components are not taken into account, we expect to observe the predicted spread to be lower than the actual spread traded in the market.

3. Application to Real Estate Index Linked Swaps

In this section we apply the model developed above to determine TRS fair spread for the real estate index linked swaps, conditional on counterparty default risk, and discuss the results. We price TRS fair spread using a sample data drawn from IPD indices, different maturities (1, 3 and 5 years), and different counterparty default probabilities. The IPD monthly and annual indices used in our analysis measure returns to direct investment in commercial property. The indices are compiled from valuations and management records for individual buildings in complete portfolios, collected directly from investors by IPD. All valuations used in the indices are conducted by qualified valuers, independent of the property owners or managers, working to RICS guidelines. The indices show total returns on capital employed in market standing investments. Standing investments are properties held from one valuation period to the next. The market results exclude any properties bought, sold, under development, or subject to major refurbishment in the course of the month. The monthly and annual results are chain-linked into a continuous, time-weighted, index series. Total return is overall return on capital employed, and is the sum of income return and capital growth. Income return is income receivable net of property management and irrecoverable costs divided by capital employed through the month. Capital growth is change in capital value from one valuation to the next net of any capital flows, divided by capital employed. Capital employed is capital value at the start of the month plus half of any net capital flow, minus half of income receivable (that is, the calculation assumes flows of capital and reinvested income are even through the month). Rental value growth is synonymous with estimated rental value growth and open market rental value growth. It is the

percentage change in the rental value used in the valuation from one month end to the next.

We use LIBOR rate as the market interest rate, with maturities up to twelve months denoted as²: $L(t_{k-1}, t_k) = [\exp(r\Delta)-1] / \Delta$

In the absence of counterparty financial information, we cannot price a particular TRS using equation (8). Therefore, each TRS fair spread is computed using term structure of default probabilities (see Table 1), per each credit rating class, provided by Moody's Report (2005). Within this framework, and to simplify matters, default probability is not time varying, it only varies with TRS maturity, as shown in Table 1. We use Moody's term structure of default probabilities in equation (10) to compute hazard rate, h , of each credit rating class over different time horizon. Default probabilities over each Δ , given \mathcal{F}_t , are computed using equation (9).

TRS fair spread, δ_{TRS} , is computed as a weighted average of contract swaplets' fair spreads. This spread is computed using

$$\delta_{\text{TRS}} = \frac{\int_k^n \delta_k e^{\int_k^n r_s ds}}{e^{\int_k^n r_s ds} + 1} \quad (11)$$

where δ_k is the swaplet's spread active between t_k and t_{k-1} and paid at t_k .

In this setting, TRS fair spread depends largely on the volatility of the underlying real estate index. Since some real estate indices are re-valuated monthly and others annually, we present standard descriptive statistics separately (see Table 2 and 3, respectively).

² British Bankers Association website: www.bba.org.uk

We also test serial dependence of the indices, for which we have monthly data. According to the Ljung-Box statistics for returns, there is first order and higher order autocorrelation, significant at the 1 percent level, meaning that the series is serially correlated. The Augmented Dickey-Fuller test (ADF) allows us to conclude that, at the 5 percent significance level, the series are stationary. We present the ARMA (p, q) processes that minimize Schwarz criterion.

Table 2 and 3 exhibit relevant information supporting the attractiveness of property derivatives. In the UK, IPD real estate indices have relatively low risk with corresponding high-risk adjusted return. For example, All Property Income Return Index earned in average 6.6 percent/year, from 1980 to 2004, with only 1 percent of volatility. Even the simpler performance measures, such as Sharpe ratio, reveal the extraordinary performance and the attractiveness of real estate indices. The low volatility permits us to infer low TRS spreads. Table 4 reports the TRS spreads for the IPD monthly index, and Table 5, 6 and 7, report the spreads for the annual indices. Since each TRS is a sum of swaplets, TRS fair spread is computed using equation (11). Table 4 reports TRS fair spreads (basis points) of Total Return and Capital Value Indices for different levels of default risk. The volatility of index returns is computed using a rolling window of the last 3 years of monthly observations. For these indices we consider that cash-flows are paid semi-annually, meaning that swaplets have a 6 month maturity. Three key features are worth noting here:

- TRS spreads of IPD Office Indices are higher than those of IPD All Property Indices. This can be explained by the higher volatility of the former index.
- For investment-grade rated investor, TRS fair spread increases with TRS maturity, because default intensity also increases with time, meaning that for

these investors, the probability of bankruptcy over the period $[t_k, t_{k-1}]$ is greater than the probability of bankruptcy over the previous period.

- For a speculative-grade (B and Caa-C) rated investor, TRS fair spread decreases with TRS maturity, because default intensity also decreases with time, meaning that the probability of bankruptcy over the period $[t_k, t_{k-1}]$ is lower than the one over the previous period.

Table 5 reports TRS fair spreads (basis points) of office, retail and industrial sectors indices for different levels of default risk. The volatility of index returns is computed using a rolling window of the last 13 years of yearly observations. Spreads are computed assuming swaplets with one-year maturity. The results can be summarised as follows:

- TRS of Income Return Indices, for all sectors and maturities, have the lowest spreads reflecting the low volatility of these indices.
- Overall, TRS of Office Indices have the highest spreads.
- The pattern observed earlier for investment-grade and speculative-grade rated investors is also observed for these indices.

Table 6 and 7 report TRS fair spreads for Capital Growth and Rental Value Indices of Offices, by region, respectively. Once again, TRS spreads are function of volatility that is computed using a rolling window of the last 13 years of yearly observations; overall, Rental Value Index is less riskier than Capital Growth Index for the Office sector.

To the best of our knowledge, no TRS market quotes are available for these indices over the sample period studied here. Tullet Prebon Corporation have available indicative

swap prices for a range of property derivative contracts, with different maturities. For example, the average spread, over the period 11/05 to 03/06, of a LIBOR – IPD UK All Property swap with 1 year of maturity, is around 400 basis points (bp). The lowest spread is around 100bp and the average spread for the several LIBOR – IPD swaps is around 300bp. Whilst our sample period is different, Tullet Prebon market quotes are far higher than the spread of around 65bp observed in our results. However, as explained earlier, our results are based on the low volatility reported in Table 2 and 3. Under a scenario analysis of LGD equal to 100 percent and the index return volatility of around 30 percent, the fair spread of a 1 year TRS done with a counterparty rated with a Caa-C rating is around 140bp. Thus, assuming that the market spreads are nearly fair, the model only explains a small percentage of the market spread. Such differences can be attributed to factors other than counterparty credit risk, such as liquidity and transaction costs. Given the high illiquidity, high transaction costs and relatively low turnover in real estate asset markets, a broker that trades this kind of derivatives must charge a liquidity premium. Notwithstanding, our results are in accordance with the evidence presented in Buttimer et al. (1997), who reported that the spread over LIBOR was 0.125 bp on a US real estate index TRS contract between Morgan Stanley and a pension fund trying to reduce its exposure to real estate assets.

Other factors can also explain the differences between the market and the predicted spreads. These are related to market microstructure theory. Brokers, who trade property derivatives and have to cover their exposure to property through a hedging strategy, incur high inventory holding and adverse selection costs³, a consequence of illiquidity

³ Inventory holding costs arise because dealers have additional costs by carrying undesirable long or short inventory positions. This unbalance is due to temporal divergences between buy and sell orders and moreover by the obligation to provide liquidity. The spread would then arise as a mechanism to keep the inventory at a desirable level. Adverse selection costs arise because dealers might face traders with

in property asset markets. This issue deserves a further investigation in a future study within microstructure theory, which is now well developed field of investigation (see O'Hara (1995)).

4. Conclusion

In the UK, the two main types of property derivatives currently available are property index certificates and total return swaps based on IPD indices. ABN AMRO bank led the recent attempt to provide two-way prices on total return swaps based on all property and sector based IPD indices. A TRS is a bilateral contract between a total return payer, who owns the asset, and a total return receiver, who will enjoy the asset's cash flows or returns without owning it. In this paper we extend the existing TRS valuation models to incorporate counterparty default risk and demonstrate that the higher the volatility of returns, and counterparty default risk, the higher the spread over market interest rate. Our results show that computed spreads on IPD indices are much lower compared to a sample of quotes we obtained from one of the traders in the market. Based on our analysis, we conclude that the under estimation arises because the market spread incorporates other components besides counterparty default risk including lack of transparency, low market liquidity and high transaction costs in underlying property asset markets. In practice, TRS traders assume additional exposures when they trade property derivatives because of lack of information, transparency and thin trading volume in the property markets.

superior information, which force the dealer to set the spread in order to maximize the difference between the gains obtained from trading with liquidity motivated traders and the losses from trading with informed traders.

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Figure 1 – Total Return Swap’s Cash-Flows

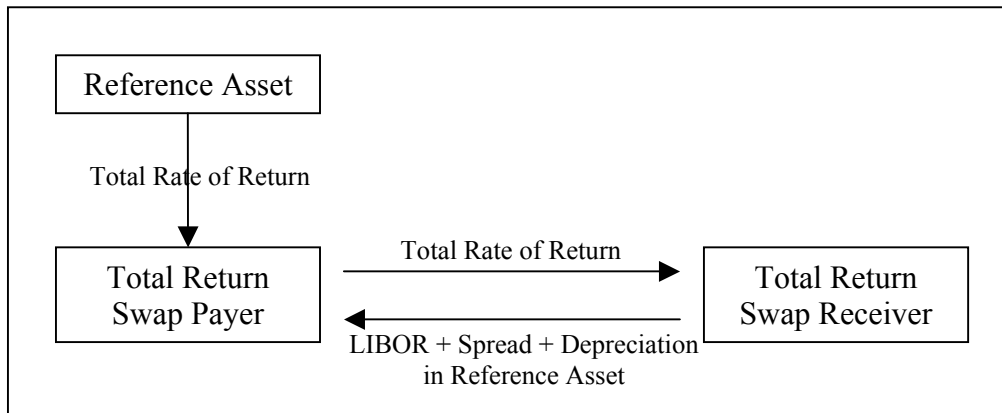


Table 1 – Default Probabilities’ Term Structure, per Credit Rating Class

Rating	1 year	3 years	5 years
Aaa	0.0000	0.0002	0.0019
Aa	0.0006	0.0032	0.0078
A	0.0008	0.0054	0.0122
Baa	0.0031	0.0169	0.0340
Ba	0.0139	0.0548	0.0993
B	0.0456	0.1524	0.2380
Caa-C	0.1507	0.3182	0.4050

Moody’s Report (2005)

Table 2 – Descriptive Statistics (monthly data, 1991-2001)

	Capital Value IPD		Total Return IPD	
	All Property	Office	All Property	Office
Mean ⁺	1.10%	-0.53%	8.93%	7.74%
Volatility ⁺	2.22%	2.73%	2.21%	2.73%
LB Q(1)*	98.1	101.2	97.8	100.8
LB Q(12)*	369.6	493.6	363.7	477.2
ADF (12)**	-3.1	-2.9	-3.2	-3.0
ARMA (p, q)	(2, 2)	(2, 1)	(2, 2)	(2, 2)

⁺ Annualised data. LB Q(L) is the Ljung-Box test for returns, using L lagged observations. ADF (L) is the Augmented Dickey-Fuller test. The ADF 5% critical value is -2.8859. ARMA (p, q) model is selected using Schwarz criterion.

* significant at the 1 percent level; ** significant at the 5 percent level.

Table 3 – Descriptive Statistics (yearly data, 1980-2004)

Sector	Index	Mean	Volatility
All Property	Capital Growth Index	3.5%	8.0%
	Income Return Index	6.6%	0.9%
	Rental Value G. Index	3.8%	7.6%
	Total Return Index	10.0%	7.7%
Retail	Capital Growth Index	5.3%	6.9%
	Income Return Index	5.9%	0.8%
	Rental Value G. Index	5.4%	5.1%
	Total Return Index	11.1%	6.6%
Office	Capital Growth Index	2.0%	9.9%
	Income Return Index	6.8%	1.1%
	Rental Value G. Index	2.6%	11.0%
	Total Return Index	8.7%	9.5%
Industrial	Capital Growth Index	3.0%	8.2%
	Income Return Index	8.3%	0.9%
	Rental Value G. Index	3.2%	6.8%
	Total Return Index	11.3%	8.1%
Other Property	Capital Growth Index	3.8%	6.7%
	Income Return Index	5.2%	0.8%
	Rental Value G. Index	2.9%	3.4%
	Total Return Index	9.0%	7.0%

Table 4 – Average TRS' spread (b.p. - monthly data, 1994-2001)

	Total Return IPD						Capital Value IPD					
	All Property			Office			All Property			Office		
	1	3	5	1	3	5	1	3	5	1	3	5
Aaa	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001
Aa	0.001	0.002	0.003	0.002	0.002	0.003	0.001	0.002	0.003	0.002	0.002	0.003
A	0.002	0.003	0.004	0.002	0.004	0.005	0.002	0.003	0.004	0.002	0.003	0.005
Baa	0.006	0.010	0.013	0.008	0.011	0.015	0.006	0.010	0.012	0.008	0.011	0.014
Ba	0.028	0.032	0.038	0.037	0.037	0.044	0.028	0.032	0.037	0.036	0.036	0.042
B	0.092	0.092	0.096	0.120	0.107	0.111	0.091	0.090	0.094	0.116	0.102	0.107
Caa-C	0.296	0.201	0.176	0.386	0.234	0.205	0.293	0.197	0.173	0.373	0.225	0.196

TRS' fair spread is computed by equation (11), using 6-month swaplets. Reported values are the average of those spreads.

Table 5 – Average TRS’ spread (b.p. - yearly data, 1994-2004)

Maturity: 1 year								
Sector	Index	Credit Rating						
		Aaa	Aa	A	Baa	Ba	B	Caa-C
All Property	Capital Growth Index	0.00	0.12	0.16	0.63	2.85	9.52	33.31
	Income Return Index	0.00	0.02	0.02	0.08	0.37	1.22	4.28
	Rental Value G. Index	0.00	0.12	0.16	0.61	2.73	9.10	31.83
	Total Return Index	0.00	0.12	0.16	0.61	2.77	9.24	32.35
Retail	Capital Growth Index	0.00	0.10	0.14	0.53	2.37	7.90	27.65
	Income Return Index	0.00	0.01	0.02	0.07	0.34	1.12	3.93
	Rental Value G. Index	0.00	0.08	0.10	0.39	1.77	5.91	20.69
	Total Return Index	0.00	0.10	0.13	0.51	2.31	7.70	26.96
Office	Capital Growth Index	0.00	0.15	0.20	0.78	3.54	11.80	41.29
	Income Return Index	0.00	0.02	0.02	0.09	0.43	1.42	4.97
	Rental Value G. Index	0.00	0.17	0.22	0.85	3.85	12.85	44.97
	Total Return Index	0.00	0.15	0.20	0.76	3.43	11.45	40.07
Industrial	Capital Growth Index	0.00	0.12	0.16	0.62	2.81	9.36	32.77
	Income Return Index	0.00	0.01	0.02	0.07	0.32	1.08	3.78
	Rental Value G. Index	0.00	0.11	0.14	0.55	2.47	8.22	28.78
	Total Return Index	0.00	0.12	0.16	0.61	2.77	9.24	32.32
Other Property	Capital Growth Index	0.00	0.09	0.12	0.48	2.15	7.16	25.04
	Income Return Index	0.00	0.01	0.01	0.05	0.23	0.78	2.73
	Rental Value G. Index	0.00	0.05	0.06	0.24	1.10	3.67	12.84
	Total Return Index	0.00	0.09	0.13	0.49	2.19	7.31	25.58

Maturity: 3 year								
Sector	Index	Credit Rating						
		Aaa	Aa	A	Baa	Ba	B	Caa-C
All Property	Capital Growth Index	0.01	0.22	0.38	1.18	3.82	10.62	22.08
	Income Return Index	0.00	0.03	0.05	0.15	0.49	1.37	2.84
	Rental Value G. Index	0.01	0.21	0.36	1.12	3.65	10.13	21.06
	Total Return Index	0.01	0.22	0.37	1.14	3.71	10.31	21.43
Retail	Capital Growth Index	0.01	0.19	0.31	0.98	3.17	8.80	18.30
	Income Return Index	0.00	0.03	0.04	0.14	0.45	1.26	2.61
	Rental Value G. Index	0.01	0.14	0.24	0.74	2.40	6.67	13.86
	Total Return Index	0.01	0.18	0.30	0.95	3.09	8.58	17.84
Office	Capital Growth Index	0.02	0.28	0.46	1.45	4.72	13.11	27.25
	Income Return Index	0.00	0.03	0.06	0.18	0.58	1.60	3.32
	Rental Value G. Index	0.02	0.30	0.50	1.57	5.10	14.17	29.45
	Total Return Index	0.02	0.27	0.45	1.41	4.57	12.70	26.41
Industrial	Capital Growth Index	0.01	0.22	0.37	1.17	3.80	10.55	21.93
	Income Return Index	0.00	0.02	0.04	0.13	0.42	1.18	2.45

	Rental Value G. Index	0.01	0.19	0.33	1.03	3.34	9.27	19.27
	Total Return Index	0.01	0.22	0.37	1.15	3.74	10.39	21.60
	Capital Growth Index	0.01	0.17	0.28	0.88	2.86	7.94	16.50
Other	Income Return Index	0.00	0.02	0.03	0.09	0.30	0.85	1.76
Property	Rental Value G. Index	0.01	0.08	0.14	0.45	1.44	4.01	8.34
	Total Return Index	0.01	0.17	0.29	0.90	2.92	8.12	16.87

Maturity: 5 year

Sector	Index	Credit Rating						
		Aaa	Aa	A	Baa	Ba	B	Caa-C
All Property	Capital Growth Index	0.08	0.33	0.52	1.46	4.33	10.70	18.98
	Income Return Index	0.01	0.04	0.07	0.19	0.56	1.38	2.44
	Rental Value G. Index	0.08	0.32	0.50	1.40	4.13	10.22	18.13
	Total Return Index	0.08	0.32	0.51	1.42	4.19	10.36	18.39
Retail	Capital Growth Index	0.07	0.28	0.43	1.21	3.57	8.84	15.68
	Income Return Index	0.01	0.04	0.06	0.17	0.51	1.27	2.25
	Rental Value G. Index	0.05	0.21	0.33	0.93	2.74	6.78	12.02
	Total Return Index	0.07	0.27	0.42	1.17	3.48	8.60	15.25
Office	Capital Growth Index	0.10	0.41	0.64	1.80	5.33	13.18	23.39
	Income Return Index	0.01	0.05	0.08	0.22	0.65	1.61	2.86
	Rental Value G. Index	0.11	0.44	0.69	1.94	5.74	14.20	25.18
	Total Return Index	0.10	0.40	0.62	1.74	5.16	12.75	22.62
Industrial	Capital Growth Index	0.08	0.33	0.52	1.46	4.31	10.66	18.91
	Income Return Index	0.01	0.04	0.06	0.16	0.47	1.17	2.08
	Rental Value G. Index	0.07	0.29	0.46	1.28	3.79	9.36	16.61
	Total Return Index	0.08	0.33	0.51	1.43	4.24	10.49	18.60
Other Property	Capital Growth Index	0.06	0.25	0.39	1.09	3.22	7.96	14.12
	Income Return Index	0.01	0.03	0.04	0.11	0.34	0.84	1.49
	Rental Value G. Index	0.03	0.12	0.19	0.54	1.61	3.98	7.05
	Total Return Index	0.06	0.25	0.40	1.11	3.29	8.14	14.45

TRS' fair spread is computed by equation (11), using 1-year swaplets. Reported values are the average of those spreads.

Table 6 – Average TRS' spread on Capital Growth Offices Index by Region (b.p. - yearly data, 1994-2004)

Region	Maturity	Credit Rating						
		Aaa	Aa	A	Baa	Ba	B	Caa-C
All Office	1	0.00	0.15	0.20	0.78	3.54	11.80	41.29
	3	0.02	0.28	0.46	1.45	4.72	13.11	27.25
	5	0.10	0.41	0.64	1.80	5.33	13.18	23.39
City	1	0.00	0.18	0.24	0.93	4.19	13.98	48.91
	3	0.02	0.32	0.55	1.71	5.56	15.44	32.10
	5	0.12	0.48	0.76	2.12	6.27	15.51	27.52
Mid	1	0.00	0.20	0.26	1.02	4.59	15.32	53.61

Town	3	0.02	0.36	0.60	1.88	6.11	16.98	35.30
	5	0.13	0.53	0.83	2.33	6.90	17.06	30.27
West End	1	0.00	0.20	0.27	1.03	4.64	15.48	54.19
	3	0.02	0.36	0.61	1.90	6.17	17.16	35.66
	5	0.13	0.54	0.84	2.36	6.97	17.24	30.58
Central London Fringe	1	0.00	0.18	0.24	0.95	4.27	14.24	49.83
	3	0.02	0.33	0.56	1.74	5.66	15.72	32.67
	5	0.12	0.49	0.77	2.15	6.36	15.73	27.91
Outer London	1	0.00	0.14	0.18	0.70	3.18	10.59	37.06
	3	0.02	0.25	0.42	1.31	4.24	11.77	24.46
	5	0.09	0.37	0.58	1.61	4.78	11.81	20.95
South East	1	0.00	0.12	0.16	0.62	2.79	9.29	32.52
	3	0.01	0.22	0.37	1.14	3.71	10.31	21.43
	5	0.08	0.32	0.50	1.41	4.17	10.30	18.28
South West	1	0.00	0.13	0.17	0.67	3.02	10.06	35.22
	3	0.02	0.24	0.41	1.27	4.11	11.42	23.74
	5	0.09	0.36	0.56	1.58	4.68	11.56	20.51
Eastern	1	0.00	0.14	0.18	0.72	3.23	10.78	37.72
	3	0.02	0.25	0.43	1.34	4.33	12.04	25.02
	5	0.09	0.38	0.59	1.66	4.90	12.11	21.49
East Midlands	1	0.00	0.12	0.16	0.60	2.72	9.06	31.72
	3	0.01	0.22	0.37	1.15	3.72	10.33	21.48
	5	0.08	0.33	0.51	1.44	4.25	10.52	18.66
West Midlands	1	0.00	0.11	0.15	0.59	2.65	8.84	30.92
	3	0.01	0.21	0.36	1.12	3.63	10.08	20.95
	5	0.08	0.32	0.50	1.39	4.13	10.20	18.10
North West	1	0.00	0.10	0.13	0.52	2.33	7.75	27.13
	3	0.01	0.19	0.31	0.98	3.17	8.80	18.30
	5	0.07	0.28	0.43	1.22	3.60	8.90	15.79
Yorks & Humber	1	0.00	0.12	0.16	0.63	2.84	9.47	33.15
	3	0.01	0.23	0.38	1.19	3.86	10.74	22.32
	5	0.08	0.34	0.53	1.49	4.40	10.88	19.30
North East	1	0.00	0.11	0.14	0.54	2.46	8.19	28.66
	3	0.01	0.20	0.33	1.03	3.34	9.29	19.32
	5	0.07	0.29	0.46	1.28	3.79	9.36	16.61
Scotland	1	0.00	0.12	0.15	0.60	2.69	8.96	31.36
	3	0.01	0.21	0.36	1.12	3.63	10.09	20.98
	5	0.08	0.32	0.50	1.39	4.11	10.16	18.03
Wales	1	0.00	0.10	0.14	0.53	2.41	8.04	28.12
	3	0.01	0.19	0.32	1.00	3.24	9.01	18.73
	5	0.07	0.28	0.44	1.24	3.66	9.04	16.04

TRS' fair spread is computed by equation (11), using 1-year swaplets. Reported values are the average of those spreads.

Table 7 – Average TRS’ spread on Rental Value Offices Index by Region (b.p. - yearly data, 1994-2004)

Region	Maturity	Credit Rating						
		Aaa	Aa	A	Baa	Ba	B	Caa-C
All Office	1	0.00	0.17	0.22	0.85	3.85	12.85	44.97
	3	0.02	0.30	0.50	1.57	5.10	14.17	29.45
	5	0.11	0.44	0.69	1.94	5.74	14.20	25.18
City	1	0.00	0.22	0.30	1.16	5.23	17.43	61.00
	3	0.03	0.40	0.68	2.12	6.86	19.06	39.62
	5	0.14	0.59	0.93	2.60	7.70	19.04	33.78
Mid Town	1	0.00	0.24	0.32	1.23	5.53	18.43	64.50
	3	0.03	0.42	0.72	2.24	7.27	20.21	42.01
	5	0.15	0.63	0.98	2.76	8.16	20.17	35.78
West End	1	0.00	0.23	0.30	1.17	5.29	17.63	61.70
	3	0.03	0.41	0.69	2.15	6.98	19.39	40.31
	5	0.15	0.60	0.95	2.65	7.84	19.39	34.40
Central London Fringe	1	0.00	0.21	0.28	1.09	4.93	16.43	57.51
	3	0.02	0.38	0.64	2.00	6.49	18.04	37.50
	5	0.14	0.56	0.88	2.46	7.27	17.98	31.90
Outer London	1	0.00	0.15	0.20	0.78	3.51	11.70	40.94
	3	0.02	0.27	0.46	1.43	4.65	12.92	26.86
	5	0.10	0.40	0.63	1.76	5.22	12.90	22.89
South East	1	0.00	0.12	0.16	0.63	2.86	9.54	33.40
	3	0.01	0.22	0.37	1.17	3.79	10.53	21.89
	5	0.08	0.33	0.51	1.44	4.26	10.52	18.66
South West	1	0.00	0.13	0.17	0.65	2.93	9.78	34.21
	3	0.01	0.23	0.39	1.24	4.01	11.13	23.14
	5	0.09	0.35	0.55	1.54	4.56	11.27	19.99
Eastern	1	0.00	0.16	0.22	0.85	3.84	12.80	44.78
	3	0.02	0.30	0.50	1.58	5.11	14.21	29.53
	5	0.11	0.44	0.69	1.94	5.74	14.20	25.19
East Midlands	1	0.00	0.10	0.13	0.51	2.32	7.74	27.09
	3	0.01	0.19	0.32	1.00	3.23	8.98	18.66
	5	0.07	0.28	0.45	1.25	3.69	9.13	16.20
West Midlands	1	0.00	0.13	0.17	0.65	2.95	9.84	34.42
	3	0.01	0.24	0.40	1.26	4.09	11.37	23.65
	5	0.09	0.36	0.56	1.58	4.67	11.54	20.47
North West	1	0.00	0.09	0.13	0.49	2.21	7.38	25.82
	3	0.01	0.18	0.30	0.94	3.05	8.46	17.59
	5	0.06	0.27	0.42	1.17	3.46	8.54	15.16
Yorks & Humber	1	0.00	0.10	0.14	0.53	2.40	8.00	27.99
	3	0.01	0.19	0.33	1.03	3.33	9.26	19.24
	5	0.07	0.29	0.46	1.28	3.80	9.39	16.67

North East	1	0.00	0.09	0.11	0.44	1.99	6.62	23.18
	3	0.01	0.16	0.27	0.84	2.73	7.59	15.77
	5	0.06	0.24	0.37	1.04	3.08	7.62	13.53
Scotland	1	0.00	0.11	0.15	0.57	2.56	8.52	29.82
	3	0.01	0.20	0.34	1.07	3.48	9.68	20.11
	5	0.07	0.30	0.48	1.33	3.94	9.74	17.28
Wales	1	0.00	0.11	0.14	0.55	2.48	8.28	28.97
	3	0.01	0.20	0.33	1.05	3.39	9.43	19.60
	5	0.07	0.30	0.46	1.29	3.83	9.47	16.80

TRS' fair spread is computed by equation (11), using 1-year swaptlets. Reported values are the average of those spreads.