Why the Data Tell Us Nothing about Returns to Scale and Externalities to Capital

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Abstract: It has long been shown that, because of the aggregation problem and the Cambridge Capital Theory Controversies, the aggregate production function cannot theoretically exist. Nevertheless, the concept is still widely and uncritically used, presumably because it gives good statistical fits to the data with plausible results. It is shown that this occurs because of the existence of an underlying accounting identity. A suitable mathematical transformation of this identity ensures that it is always possible to specify an "aggregate production function" where the putative output elasticities equal the factor shares, even though the aggregate production does not exist. This is illustrated by reference to a simulation exercise by Felipe and McCombie (2006) and a study by Oulton and O'Mahony (1994). The latter reject the hypothesis that capital is "special", in that their regression estimates demonstrate that the "output elasticity" of capital does not significantly differ from its factor share. However, it is shown in this paper why the data could not have given any other result.

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INTRODUCTION

The concept of the aggregate production function is at the heart of neoclassical growth theory and, indeed, of most of neoclassical macroeconomics. Yet, for well over half a century it has been known that the aggregate production function cannot theoretically exist, even as an approximation. This nihilistic conclusion results from the so-called "aggregation debate" which considers the conditions under which micro-production functions can be aggregated to give a well-behaved aggregate production function.[2] Fisher (2005, p.489-490)[3], who has probably done more work than most on this problem, summarised the implications as follows.

> Briefly, an examination of the conditions required for aggregation yields results such as:
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> - Except under constant returns, aggregate production functions are unlikely to exist at all.
> - Even under constant returns, the conditions for aggregation are so very stringent as to make the existence of aggregate production functions in real economies a non-event. This is true not only for the existence of an aggregate capital stock but also for the existence of such constructs as aggregate labor or even aggregate output.
> - One cannot escape the force of these results by arguing that aggregate production functions are only approximations. While, over some restricted range of the data, approximations may appear to fit, good approximations to the true underlying technical relations require close approximation to the stringent aggregation conditions, and this is not a sensible thing to suppose.

Further problems arise from the Cambridge Capital Theory Controversies of the 1960s and 1970s, although the issue was first given prominence by Joan Robinson (1953-54). This showed clearly how none of the results of the "neoclassical parable" held once one moved out of a one-commodity world (Cohen and Harcourt, 2003a). The two critiques are related, although Cohen and Harcourt (2003b, p. 232) argue that "the aggregation debate is a development *within* neoclassical theory and its applications, whereas much of the Cambridge, England, critique is from *without,* regarding the basic neoclassical intuition, robustness in more general models and appropriate methods".

[2] For a recent survey, see Felipe and Fisher (2003)

[3] See also Fisher (1992).

Nevertheless, both critiques serve to show just how flimsy are the foundations of the aggregate production function.

While both these criticisms were briefly acknowledged in textbooks and surveys in the 1970s, any reference to them has now completely disappeared from the current literature. This is notwithstanding that there has been no convincing refutation of the criticisms – at least we have yet to see any. The criticisms have simply been assumed away or ignored. Textbooks and surveys that did include a discussion of the aggregation problem and the Capital Controversies include Wan (1971), Nadiri (1970), Jones (1974), and Hacche (1979).[4] Yet there is no mention of them in later textbooks and surveys such as Maddison (1987), Barro and Sala-i-Martin (1995), Valdés (1990), Jones (1998), Aghion and Howitt (1998), and Weil (2004).[5]

So why is the aggregate production function so widely and uncritically used? The answer seems to involve a form of Friedman's (1951) methodological instrumentalism. All theories, so the argument goes, involve heroic abstraction and unrealistic assumptions, but what matters is their predictive ability. The aggregate production function passes this test with flying colours. The problem with this defence is that the estimation of a putative aggregate production function cannot provide any valid inferences about the values of the parameters of the production function (i.e., output elasticities and the aggregate elasticity of substitution) or the rate of technical change. This is because, empirically, constant-price monetary data have to be used as measures for output and capital and an underlying accounting identity precludes any meaningful estimation or test of an aggregate production function.

The implications are far reaching. The existence of the constant-price value accounting identity implies that, through a suitable mathematical transformation of this identity, any estimation of a putative aggregate production can be made to give a perfect fit to the data. The results *must* show supposed constant returns to scale and their output elasticities equalling their respective factor shares. This will occur even though the aggregate production function undoubtedly does not exist and, for example, individual firms may be subject to substantial returns to scale and subject to oligopolistic competition.

[4] Hacche (1979) did not include a discussion about the aggregation problems.
[5] There is a passing mention to the aggregation problem in Temple (1999).

This is not a new critique, but first came to prominence buried in Phelps Brown's (1957) criticism of Douglas's cross-industry regression results (see, for example, Douglas, 1948).[6] But rudimentary elements of it can be traced back to Bronfenbrenner (1944) and Marshak and Andrews (1944). The critique was later formalised by Simon and Levy (1963) and Shaikh (1974, 1980, 1987) generalised it to time-series estimation of production functions. Simon (1979a) also considered the criticism in the context of both cross-section and time-series data and thought it serious enough to mention it in his Nobel Prize lecture (Simon, 1979b). The criticism was revived and re-examined and extended by Felipe and McCombie in a number of papers. See Felipe and Adams (2005), Felipe and McCombie (2001, 2003, 2005 a&b, 2006, 2007), Felipe (2001 a&b), Felipe and Holz (2001), McCombie (1987, 1998 a&b, 2000, 2000-2001, 2001), McCombie and Dixon (1991) and McCombie and Thirlwall (1994). The critique as applied to cross-section data was also "rediscovered" by Samuelson (1979).

While Cramer (1969), Wallis (1973) and Intriligator (1978) in their econometric textbooks and Walters (1966) in his survey on production and cost functions have mentioned the argument, none pushed it to its logical conclusion: namely, that it invalidated any attempt to test or estimate the aggregate production function, *per se*. (See McCombie, 1998a, for a discussion.) Solow (1974, 1987), it is true, did attempt refutations of a couple of aspects of the critique, but these are not compelling (Shaikh, 1980, Felipe and McCombie, 2005a).

Nevertheless, in what has been seen as an important study, Oulton and O'Mahony (1994, Chapter 7) putatively test the hypothesis of the existence of increasing returns using growth data and the production function approach for UK manufacturing industries for various sub-periods over 1954-1986. Their results rejected the null hypothesis that there are externalities to capital as they also found that the estimated output elasticity of capital did not significantly differ from its factor share. Indeed, this was true of the other inputs.

The conclusions of this research have been cited on a number of occasions by, for example, Crafts *et al.* as having important policy implications. They argue that

[6] Saying that it came to prominence is perhaps a little over enthusiastic as the argument hardly attracted any attention at the time and has only been cited in a handful of papers since. But that is true of the critique in general.

"it seems that for physical capital these [externalities] are trivial (Oulton and O'Mahony, 1994)", (Crafts and Toniolo, p.1996, p.32); "moreover, recent work at the NIESR has found no evidence that social returns were significantly larger than private returns to fixed capital formation in British Manufacturing (Oulton and O'Mahony, 1994)" (Bean and Crafts, 1996, p.136); "Oulton and O'Mahony (1994), in an econometric analysis of British manufacturing during 1954-86, found that there was no support for the hypothesis that weighting capital by profits share underestimates capital's role in growth…" (Crafts, 1996, p.38).

However, in this paper we show why the data could not have failed to reject the null hypothesis that there are externalities to capital. Consequently, Oulton and O'Mahony's (1994) regressions can shed no light on this issue.

The paper is structured as follows. We begin by briefly recapitulating the general argument as to why the existence of an underlying identity precludes the estimation of an aggregate production function. We then discuss one of Felipe and McCombie's (2006) simulation exercises where they demonstrate how the estimation of a cross-industry production function using value data must give constant returns to scale, even though it is known that the individual production functions are subject to increasing returns to scale. Finally, in the light of these arguments we turn to Oulton and O'Mahony's results and show why they had to find the results that they did.

AGGREGATE PRODUCTION FUNCTIONS AND THE ACCOUNTING IDENTITY

In neoclassical production theory, the production function in its most general form is written as:

$$Q_t = f(K_t, L_t, t) \tag{1}$$

where Q, K, L, and t are output, capital, labour, and a time trend that acts as a proxy for technical change. Theoretically, Q and K should be measured in homogenous *physical* units, as equation (1) is a technological relationship. It may be expressed in growth rates as:

$$\hat{Q}_t = \lambda_t + \alpha_t \hat{K}_t + \beta_t \hat{L}_t \tag{2}$$

where α and β are the output elasticities of capital and labour and λ is the rate of technical change. The symbol ^ above a variable denotes a growth rate.

If there are constant returns to scale, perfect competition, and firms are paid their marginal products, then it can be simply shown that the following holds:

$$\hat{Q}_t = \lambda_t + a_t \hat{K}_t + (1 - a_t)\hat{L}_t \tag{3}$$

where a is capital's factor share and $(1\text{-}a)$ is the share of labour, and $a = \alpha$ and $(1\text{-}a) = (1\text{-}\alpha)$.

From Euler's theorem, output may be written as:

$$Q_t = f_{Kt}K_t + f_{Lt}L_t = r'_t K_t + w'_t L_t \tag{4}$$

where r' is the price of each machine and w' is the wage rate, both measured in commodity terms. Equation (4) expressed in growth rates is:

$$\hat{Q}_t = a_t \hat{r}'_t + (1 - a_t)\hat{w}'_t + a_t \hat{K}_t + (1 - a_t)\hat{L}_t \tag{5}$$

But, as we noted above, empirically, constant-price monetary data have to be used for output and the capital stock and it is here that an insurmountable difficulty arises. Equation (1) using these data becomes:

$$V_t = f\left(J_t, L_t, t\right) \tag{6}$$

where V and J are constant-price value added and the constant-price monetary value of capital stock, respectively.[7]

From the national accounts, the following identity must always hold:

$$V_t \equiv r_t J_t + w_t L_t \equiv \Pi_t + W_t \tag{7}$$

[7] We use V and J for output and capital measured in monetary values and Q and K for when they are measured using homogenous physical units.

where r is the rate of profit (a pure number)[8] and w is the average real wage rate measured in monetary terms. V is value added and J is the value of the capital stock, both measured in constant prices. J is usually calculated by the perpetual inventory method. In other words, the sum of total profits (Π) and the total compensation of labour (W) *must*, by definition, equal value added. Equation (7) can be written in growth rates as:

$$\hat{V}_t \equiv a_t \hat{r}_t + (1-a_t)\hat{w}_t + a_t \hat{J}_t + (1-a_t)\hat{L}_t \qquad (8)$$

But it should be noted that equation (8) does not require any of the neoclassical assumptions used to derive equation (5). Thus, equation (5), when expressed using monetary values for output and capital as equation (8), must always hold by virtue of the identity.

Neoclassical production theory generally specifies a specific functional form for equations (1) and (6), such as a Cobb-Douglas, CES, or translog production function. But this does not affect the argument. If equation (8) is integrated with respect to time, we derive the result, purely as a result of a mathematical transformation, that at time τ:

$$V_\tau \equiv B_o r_\tau^{a_\tau} w_\tau^{(1-a_\tau)} J_\tau^{a_\tau} L_\tau^{(1-a_\tau)} \qquad (9)$$

where B is the constant of integration, equal to $a_\tau^{-a_\tau}(1-a_\tau)^{-(1-a_\tau)}$. The shares are constant because only one point of time is being considered. We may illustrate this equivalence expressed in equation (9) by using data for the United Kingdom for 1990 (the exact year is immaterial), and calculating the level of output both from the identity given by equation (7) and equation (9). The results are reported in Table 1 where it can be seen that they both give exactly the same answer. Thus, at any point of time, a Cobb-Douglas will always give a good fit to the data simply as an alternative mathematical way of writing the identity.

[Table 1 about here]

[8] $r_t \equiv (V_t - w_t L_t)/J_t$

If we use cross-industry or cross-regional data and estimate $V_{it} = AJ_{it}^{\alpha} L_{it}^{\beta}$ (where i denotes the i^{th} industry or region) in logarithmic form, it follows that we should find an almost perfect fit to the extent that the variation in the logarithm of wage rate and the rate of profit is small and the factor shares do not greatly differ across observations. This is precisely what Douglas's regressions in the 1930s found, with the coefficients on capital and labour almost identical to their factor shares (Douglas, 1948). He concluded that this proved the neoclassical theory of distribution and refuted the Marxian theory (Douglas 1976), although, of course, this result is purely an artefact of the accounting identity.

Turning to time-series estimation, a stylised fact is that there is no discernible trend in the rate of profit over long periods of time and the growth of the real wage grows at roughly a constant rate. Hence, the identity given by equation (9) may be expressed as:

$$V_t \equiv r_t J_t + w_t L_t \equiv A_o e^{\lambda t} K_t^{a} L_t^{(1-a)} \qquad (10)$$

where $\lambda = (1-a)\hat{w}$. The right-hand side of equation (10) resembles the Cobb-Douglas relationship, although it is still nothing more than an alternative way of writing the accounting identity.

But, if our claim is true, why do not aggregate estimations of production functions always give good statistical fits? The fact that they do not may give the impression that they are actually behavioural equations. The poor statistical fits could be due to two reasons. First, factor shares may vary considerably over the estimation period and, secondly, the path over time of the weighted rate of profit and the wage rate may not be accurately proxied by a linear time-trend in the log-linear specification of the Cobb-Douglas (or a constant in the specification in terms of growth rates).

Empirically, the latter usually proves to be the correct explanation, and this can result in significant bias on the coefficients on the capital and labour variables. It can also be responsible for suggesting that there are increasing returns to scale. But the fit to the transformation of the identity given by, for example, the last expression in equation (10) or the translog, can always be improved by the introduction of a suitable non-linear time trend. (There is nothing in neoclassical production theory that says technical change has to be a linear function of time.) Alternatively, including a suitable capacity utilisation variable or adjusting the capital and labour inputs for the intensity of use can have the same effect. (Felipe and McCombie, 2005a).

If factor shares vary over time, then a functional form that is more flexible than the Cobb-Douglas (such as a Box-Cox transformation, which turns out to be similar to the CES) could always be used (e.g., see McCombie, 2000b, Felipe and McCombie, 2001).

FELIPE AND McCOMBIE'S SIMULATION EXERCISE

One of the most instructive ways to illustrate the problem posed by the accounting identity and the use of monetary data is through a simulation exercise, where we know both the true underlying micro-production functions in physical terms and in value terms, but the researcher only knows the latter. Felipe and McCombie (2006) used a simulation analysis to show how the estimates of a production function could be totally at variance with the actual micro-economic technology. They used cross-firm data for one year. They show that even when the firm micro-production functions display strong increasing returns to scale, the statistical estimates using monetary data must always imply constant returns to scale.

In their simulation analysis, each firm had a true Cobb-Douglas production function given by:

$$Q_i = AK_i^{\alpha}L_i^{\beta} \qquad (11)$$

where Q and K are output and the number of capital machines, both measured in *physical* units. A is the level of technology which was normalised to unity. The technological output elasticities of capital and labour were given by α = 0.9 (or 0.75 x 1.2) and β = 0.3 (or 0.25 x 1.2). It should be noted that the values of the elasticities have deliberately been chosen to be the converse of the values of the factor shares as derived from the national accounts, and multiplied by 1.20, which is the degree of increasing returns to scale. There were 10 firms and in the simulated data they had different values for Q, K, and L and a small error term was introduced to prevent perfect multicollinearity. Equation (11) was estimated using cross-firm data for two pooled periods.

To obtain data in monetary terms it was assumed that the individual firms pursue a simple constant mark-up pricing policy:

$$p_i = (1+\pi)\frac{wL_i}{Q_i} \qquad (12)$$

and, therefore, the value of output at time *t* is given by:

$$V_i \equiv p_i Q_i \equiv (1+\pi)wL_i \equiv rJ_i + wL_i \qquad (13)$$

where *w* and π are the money wage rate and the mark-up for each firm (both assumed to be constant across firms). π is taken as 0.333 which implies that capital's share in value added is $a = 1/(1+\pi) = 0.75$ and labour's share is 0.25. *J* is the constant-price monetary value of the capital stock and is calculated as:

$$J_i \equiv \frac{V_i - wL_i}{r} \qquad (14)$$

where *r* was taken to be 0.10 and *w* was assumed to be constant across firms.

The researcher only has access to the monetary, or value, data and not the physical data. When a Cobb-Douglas production function is estimated using the cross-firm hypothetical data, a very close statistical fit is found (the closeness of fit being determined solely by the error term introduced in the construction of the data). The estimate of capital's output elasticity is 0.25 (and equal to capital's share) and *not* the 'true' value of 0.9. Conversely the estimate of labour's output elasticity was 0.75 (and equal to its factor share) and not the 'true' value of 0.3. In other words, the estimates of the output elasticities are identical to the relevant factor shares and suggest that the production function exhibits constant returns to scale, even though we know that the true parameters are completely different.

In a second simulation, Felipe and McCombie show that if the true underlying firm production functions have constant returns to scale with $\alpha = 0.75$ and $\beta = (1-\alpha) = 0.25$, the estimated output elasticities are identical to those found when there are increasing returns to scale (i.e., 0.25 and 0.75 respectively). The only difference is that the value of the intercept is lower. Hence, the impact of increasing returns to scale is being captured by a supposedly higher level of technology.

Consequently, the use of value data can never be used to refute the null hypothesis of constant returns to scale, even when all firms display large returns to scale. Therefore, it comes as no surprise to learn that Oulton and O'Mahony find no evidence that there are increasing returns to UK manufacturing or that the output elasticity of capital is very close to its factor share. We turn next to a consideration of their analysis.

OULTON AND O'MAHONY'S TEST OF "IS CAPITAL 'SPECIAL' ?"

The early form of the endogenous growth theory emphasised the particular role of capital accumulation in the growth process. One of the first endogenous growth models, the so-called "linear-in-K model" or $Q = \Lambda K$ model (where Λ is a constant) assumed that the externalities associated with capital accumulation were so strong that the aggregate output elasticity of *K* (sometimes interpreted as broad capital) was unity. While this assumption is now generally accepted as being too extreme, it is still hypothesised that capital is 'special', in the sense that its output elasticity is greater than its factor share. This is because capital accumulation induces technical change. Let us assume that firm *i* has a Cobb-Douglas production function where $Q_{it} = A_0 e^{\lambda t} K_{it}^{\alpha} L_{it}^{(1-\alpha)}$ and the output elasticities equal the factor shares, $\alpha = a$ and $(1-\alpha) = (1-a)$. In other words, there is perfect competition and factors are paid their marginal products. The rate of technical change is partly determined at the industry level by the growth of the total capital stock, for example, through a learning-by-doing process (Arrow 1962):

$$\lambda_t = \tilde{\lambda} + \psi \hat{K}_t \qquad (15)$$

where $\tilde{\lambda}$ is the rate of exogenous technical change and ψ gives the extent to which the growth of the aggregate capital stock induces technical change. The growth of the industry capital stock, consequently, generates a positive externality in that a faster rate of growth of it induces a faster growth of technical change. As it is an externality, it is possible to retain the assumption of perfect competition.

Ignoring aggregation problems and summing across firms we obtain $Q_t = A_0 e^{\tilde{\lambda} t} K_t^{(\alpha+\psi)} L_t^{(1-\alpha)}$, where the output elasticity of aggregate capital ($\alpha + \psi$) exceeds its factor share (a). Oulton and O'Mahony (1994) undertook two tests to determine whether or not the coefficient on capital departed from the value of its factor share.

They used cross-industry UK manufacturing data expressed in growth rates for 124 industries and considered the period 1954-1986, broken down into 8 sub-periods, and 2 longer combinations of these sub-periods. They use gross output, rather than value added, but this does not affect in any way the criticisms of the aggregate production function discussed above.

The First Test

In the first test, Oulton and O'Mahony start with the definition of multi-factor productivity growth (MFPG) (which is what they term total factor productivity growth) "actually being measured" as:

$$MFPG_{it} = \hat{Y}_{it} - (\theta_{Jit}\hat{J}_{it} + \theta_{Lit}\hat{L}_{it} + \theta_{Mit}\hat{M}_{it}) \quad (16)$$

where the θs are the factor shares, $\hat{Y}$ is gross output and $\hat{M}$ is the growth of intermediate inputs or materials, both measured in constant-price monetary values. The other variables are as defined above. The factor shares by definition must sum to unity, i.e., $\theta_{Jit} + \theta_{Lit} + \theta_{Mit} \equiv 1$. By "actually being measured", Oulton and O'Mahony mean equation (16) is calculated using factor shares and other variables taken from the census of production and other statistical sources (see Oulton and O'Mahony, 1994, p.186).

They then assume that the "true" rate of total factor productivity growth is given by:

$$MFPG_{it}^{*} = \hat{Y}_{it} - (\theta_{Lit}^{*}\hat{L}_{it} + \theta_{Jit}^{*}\hat{J}_{it} + \theta_{Mit}^{*}\hat{M}_{it}) \quad (17)$$

where the θ*s are the "true" output elasticities of the production function, which need not necessarily equal the factor shares.

In other words, Oulton and O'Mahony assume that each industry has a well-behaved aggregate production function of the general form $Y_{it} = f(A_{it,}, J_{it}, L_{it}, M_{it})$.[9] Expressing this in growth rates gives:

$$\hat{Y}_{it} = \hat{A}_{it} + \theta^{*}_{Jit}\hat{J}_{it} + \theta^{*}_{Lit}\hat{L}_{it} + \theta^{*}_{Mit}\hat{M}_{it} \qquad (18)$$

where $A_{it} = MFPG^{*}_{it}$ is the "true" rate of multi-factor productivity growth.

If there is perfect competition and constant returns to scale, the output elasticities will equal the observed factor shares, i.e., $\theta^* = \theta$. Moreover, as we noted above, it can be further shown that if, under these assumptions, factors are paid their marginal products, the "true" growth of multi-factor productivity is given by $MFPG^{*} \equiv \hat{A}_{it} \equiv \theta_{Jit}\hat{r}_{it} + \theta_{Lit}\hat{w}_{it} + \theta_{Mit}\hat{m}_{it}$. The variable $\hat{m}$ is the growth of the relative price of intermediate inputs (materials). In other words, the rate of technical change (or total multi-factor productivity growth) is equal to the sum of the growth of real factor prices, each weighted by its factor share.

Subtracting equation (17) from equation (16) gives the equation:

$$MFPG_{it} \equiv MFPG^{*}_{it} + (\theta^{*}_{Jit} - \theta_{Jit})\hat{J}_{it} + (\theta^{*}_{Lit} - \theta_{Lit})\hat{L}_{it} + (\theta^{*}_{Mit} - \theta_{Mit})\hat{M}_{it} \qquad (19)$$

As $MFPG^{*}_{it}$, the putative correct measure of total factor productivity growth, is unobservable, Oulton and O'Mahony contend that as it differs across industries, it can be proxied by:

$$MFPG^{*}_{it} = \eta_i + \chi_t + \varepsilon_{it} \qquad (20)$$

where η varies across industries, but is constant over time, χ is constant across industries but varies over time and ε is a random error. Their estimating equation is:

9 Although the industries are at a relatively high level of disaggregation, the production functions are still "aggregate" in that the production function of any one industry uses the summed values of output and the capital stock of the individual firms in that industry.

$$MFPG_{it} = c + b_1\hat{J}_{it} + b_2\hat{L}_{it} + b_3\hat{M}_{it} + \rho_{it} \tag{21}$$

where c denotes generically the intercept term (in this case it is $E(\eta_i) + \chi_t$) and ρ is the composite error term. The coefficients are $b_1 = \theta^*_{Jit} - \theta_{Jit}$, $b_2 = \theta^*_{Lit} - \theta_{Lit}$, and $b_3 = \theta^*_{Mit} - \theta_{Mit}$. Oulton and O'Mahony estimated equation (21) using the UK cross-manufacturing data. If the coefficient on $\hat{J}$ (i.e., $b_1 = \theta^*_{Jit} - \theta_{Jit}$) is statistically significant and positive, they argue that this shows that the true output elasticity of capital is greater than its factor share. (The same is also true for $\hat{L}$ and $\hat{M}$.) They ran the regressions for the 10 sub-periods separately over the period 1954-1986 and found that estimated coefficients b_1, b_2, and b_3 were nearly always statistically insignificant. Therefore, as the $\theta^*_J s$ do not significantly differ from the $\theta_J s$, they conclude "these results therefore provide no support at all for the view that the role of capital has been understated" (p.162).

But what precisely is the interpretation of equation (21)? Recall that we are using constant-price monetary data and therefore the following identity must always hold:

$$\hat{Y}_{it} \equiv (\theta_{Jit}\hat{r}_{it} + \theta_{Lit}\hat{w}_{it} + \theta_{Mit}\hat{m}_{it}) + (\theta_{Jit}\hat{J}_{it} + \theta_{Lit}\hat{L}_{it} + \theta_{Mit}\hat{M}_{it}) \tag{22}$$

or,

$$MFPG_{it} \equiv (\theta_{Jit}\hat{r}_{it} + \theta_{Lit}\hat{w}_{it} + \theta_{Mit}\hat{m}_{it}) \tag{23a}$$

$$\equiv \hat{Y}_{it} - (\theta_{Jit}\hat{J}_{it} + \theta_{Lit}\hat{L}_{it} + \theta_{Mit}\hat{M}_{it}) \tag{23b}$$

In other words, empirically equations (22), (23a), and (23b) hold irrespective of the true underlying industry aggregate production functions, which may, in fact, not exist. The only reason that we may not find a perfect statistical fit to these equations is that the factor shares differ between the industries and over time. By manipulating the identity we obtain:

$$MFPG_{it} \equiv MFPG_{it} + (\theta_{Jit} - \theta_{Jit})\hat{J}_{it} + (\theta_{Lit} - \theta_{Lit})\hat{L}_{it} + (\theta_{Mit} - \theta_{Mit})\hat{M}_{it} \tag{24}$$

Note that all the variables are observed variables. Consequently, if *MFPG* (or the observed sum of the weighted growth of the factor prices) is uncorrelated with the growth of the factor inputs, and there is no *a priori reason* why should expect the contrary (Salter 1960), and, following Oulton and O'Mahony, we were to estimate equation (21), we should expect to find that the estimated coefficients b_1, b_2 and b_3 to be equal to zero. In other words, all this shows is that *MFPG* is orthogonal to the other regressors (i.e., $\hat{J}, \hat{L}$ and $\hat{M}$) in equation (24). Alternatively, we can simply interpret equation (21) as an auxiliary regression between the two sets of regressors in parentheses in the identity given by equation (22). It should be emphasised that all this has nothing to do with an aggregate production function, which, as we have emphasised, does not theoretically exist.

These remarks are confirmed by the results in Table 2, using data from Oulton and O'Mahony (1994). As we are merely dealing with an identity, we should not expect the results to differ greatly between the separate sub-periods and so we have pooled the sub-periods.

Equation (i) in Table 2 is nothing more than the estimation of the full identity given by equation (23b). The coefficient of $\hat{Y}_{it}$ should be unity and the coefficients of the other regressors are the (negative) average values of the factor shares. It can be seen that the estimated coefficients are close to the shares, but are not exactly the same because of the variability of the shares between industries and over time. But the point to be made is that this regression is not a *test* of a behavioural hypothesis, but merely illustrates that above argument is a question of logic.[10]

Equation (21), which Oulton and O'Mahony use to test the externality hypothesis, replaces *MFPG* in equation (24) (or, alternatively, $\hat{Y}$ in equation (23b)) by a constant. The results are reported in Table 2 as equation (ii). All the coefficients are very near to zero, which is what we would expect to find solely from the identity. (The coefficients $\hat{L}$ and $\hat{M}$ are statistically significant, but this seems to be purely coincidental. They are not usually significant when the individual sub-periods are regressed. See also the results of Oulton and O'Mahony, (1984, Table 7.1, p.162).) All that the results show is that the sum of the weighted growth of the factor prices is orthogonal to the growth of the factor inputs or, equivalently, that the growth of

[10] Equation (21) was estimated for each period separately and we found, not surprisingly, very similar results.

output is correlated with the growth of the factor inputs biasing the estimates towards zero. The former implies the latter, and *vice versa*.

We tested explicitly this expectation that the growth of output is correlated with the growth of the factor inputs by regressing $\hat{Y}$ on $\hat{K}$, $\hat{L}$, and $\hat{M}$ and estimated:

$$\hat{Y}_{it} = c + b_4 \hat{K}_{it} + b_5 \hat{L}_{it} + b_6 \hat{M}_{it} + \zeta_{it} \qquad (25)$$

where the estimated coefficients b_4, b_5 and b_6 are expected to be approximately equal to the average values of the shares, θ_J, θ_L and θ_M. The results are reported in Table 2, equation (iii). (A neoclassical economist would regard this as a direct estimate of the production function.) Because $\hat{L}$, $\hat{K}$, and $\hat{M}$ are large components of $\hat{Y}$, it is not surprising the R^2 is so high (0.790). The estimated coefficients of the growth of the factor inputs are close to their respective factor shares. However, these estimates cannot tell us anything about whether or not the putative aggregate output elasticities (which almost certainly do not exist) equal their respective shares. This test can also shed no light on the degree of returns to scale, as the identity guarantees that the estimates of the putative output elasticities will always equal the factor shares and hence sum to unity.[11] The coefficients, in fact, sum to 1.022.

[Table 2 about here]

The Second Test

Oulton and O'Mahony also proposed a second test, which is equally flawed as a test of whether capital is special. In fact, it is merely a different specification of the first test and does not really tell us anything new. They estimated the equation:

[11] This assumes, as discussed and confirmed in the text, that MFPG is orthogonal to $\hat{J}, \hat{L},$ and $\hat{M}$. However, our argument in no way depends upon this relationship. If MFPG and the variables $\hat{J}, \hat{L},$ and $\hat{M}$ were not orthogonal, the estimates of the factor shares would be biased and their sum may be statistically significantly different from unity. There might be an economic explanation for this, but it would have nothing to do with an aggregate production function. What is determining the goodness of fit, and the (biased) estimates of the coefficients (the factor shares), is still the identity, albeit misspecified by the omission of MFPG.

$$(\hat{Y}_{it} - \hat{L}_{it}) = c + \theta_{Jit}(\hat{J}_{it} - \hat{L}_{it}) + [\theta_{Jit} + \theta_{Lit} + \theta_{Mit} - 1]\hat{L}_{it} + \theta_{Mit}(\hat{M}_{it} - \hat{L}_{it}) + \xi_{it} \quad (26)$$

where ξ is the error term, using panel data methods. "If the theory underlying the calculation of MFP growth rates is correct, we would expect that the estimated coefficients on $\hat{J}_{it}$ and $\hat{M}_{it}$ in a panel regression would be approximately equal to the sample average of the value shares for capital and intermediate input respectively and that the coefficient on $\hat{L}_{it}$ would be equal to zero, since the value shares sum to one. One the other hand, if standard theory understates the role of capital and if increasing returns exist, then the sum of the elasticities exceeds one (that is $\theta_{Jit} + \theta_{Lit} + \theta_{Mit} > 1$), and coefficient on $\hat{L}_{it}$ is positive. Also, the coefficient on capital should be significantly larger than capital's value share" (Oulton and O'Mahony, 1984, p.163. Their notation has been changed to that used in this paper).

They find that the regression results "all reject the hypothesis of a special role for capital" (p.165). The coefficient on $\hat{L}_{it}$ is never statistically significant and the coefficients on $\hat{J}_{it}$ and $\hat{M}_{it}$ are very close to their average shares.

The fallacy of this interpretation may be straightforwardly shown, as the problem is that the results are once again driven by the identity. Definitionally, the following identity holds:

$$(\hat{Y}_{it} - \hat{L}_{it}) \equiv (\theta_{Jit}\hat{r}_{it} + \theta_{Lit}\hat{w}_{it} + \theta_{Mit}\hat{m}_{it}) + \theta_{Jit}(\hat{J}_{it} - \hat{L}_{it}) + 0\hat{L}_{it} + \theta_{Mit}(\hat{M}_{it} - \hat{L}_{it}) \quad (27)$$

It is likely that the sum of the weighted factor prices, i.e., $(\theta_{Jit}\hat{r}_{it} + \theta_{Lit}\hat{w}_{it} + \theta_{Mit}\hat{m}_{it})$, varies between industries (and possibly over time) and so the fixed-effects estimation of equation (26) captures this variation, when the term is dropped from the identity, equation (27).

Given the previous results, it is not surprising that Oulton and O'Mahony find the estimates of the coefficients of $(\hat{J} - \hat{L})$ and $(\hat{M} - \hat{L})$ are not significantly different from the average factor shares and the coefficient on $\hat{L}$ is not significantly different from zero.[12] These results are precisely what we would expect from the identity if the

[12] Their results are reported in Oulton and O'Mahony, (1984, Table 7.2, p.164 and Table 7.3, p.165). When they split the capital stock into plant and machinery, buildings, and vehicles, they find the

fixed effects were accurately capturing the variation of the weighted growth of factor prices across industries and time in the identity, and/or this growth rate was orthogonal to the included regressors. Indeed, estimating this regression is superfluous given the previous results. All that is being captured is the underlying identity. This is illustrated by Table 3. Equation (i) reports the full identity, where it can be seen that the coefficient on *MFPG* is slightly smaller than the predicted 1.00. Nevertheless, the estimated shares of capital (0.15) and of intermediate inputs (0.60) are very close to the average values over the 8 sub-periods (0.15 and 0.59 respectively). The coefficient of the growth of the employment is not statistically significant, which is in accord with equation (27).

[Table 3 about here]

In Table 3, the regression results of equation (ii) omit the growth of *MFP* from the identity and is the estimation of equation (27) or:

$$(\hat{Y}_{it} - \hat{L}_{it}) = c + b_7(\hat{J}_{it} - \hat{L}_{it}) + b_8\hat{L}_{it} + b_9(\hat{M}_{it} - L_{it}) + \varpi_{it} \qquad (28)$$

where ϖ_{it} is the error term.

As we know from the above results that *MFPG* is almost orthogonal to the growth of factor inputs, dropping it from the regression does not greatly bias the coefficients of the included variables, especially as we use fixed-effects panel estimation. This is confirmed by Table 3, equation (ii), where the coefficients on $\hat{J}$ and $\hat{M}$ are close to their factor shares and the growth of the labour input is again statistically insignificant. But this equation is simply a re-specification of equation (25) where $\hat{L}$ has been subtracted from both sides of the equation. The coefficients of $\hat{J}$ and $\hat{M}$ in equations (25) and (27) should each be equal (i.e., the estimates of $b_4 = b_7$ and $b_6 = b_9$). The coefficient on $\hat{L}$ should equal the estimates of $b_4 + b_5 + b_6 - 1$, which is also the case. This regression conveys no new information in addition to that implicit in

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coefficients of these variables are usually statistically insignificant. This result is probably due to the large disparities between industries in the shares in output of these three types of the capital stock, preventing any precise estimation of the average shares.

equation (25) i.e., Table 2, equation (iii). Similarly, both regressions can tell us nothing about the underlying technological conditions of production.

To summarise: the results cannot be used to infer that capital is not special, as Oulton and O'Mahony and Crafts *et al.*, do; the latter in the papers cited above. The data cannot tell us either way.

CONCLUSIONS

The conclusions of this paper may be summarised as follows.

The literature on aggregation shows that aggregate production functions do not exist in the sense that the theoretical conditions required to aggregate micro-production functions into a well-behaved aggregate production function are so stringent that in all probability actual economies do not satisfy them. Indeed, intuition would suggest that it makes little sense to aggregate the data for such diverse industries as textiles and petrochemicals and talk about the "aggregate elasticity of substitution" of this new hybrid industry.

The sole reason why the estimation of production functions using constant-price monetary data yields what may be seen as plausible results is the existence of the underlying accounting identity. If shares are roughly constant then the Cobb-Douglas "production function" may give an exceptionally good fit to the data, but the causation is from the stability of the factor shares to the Cobb-Douglas relationship, and not *vice versa*.

The underlying accounting identity ensures that it is always possible to get a good statistical fit to a constant-price monetary data production function where estimates of the "output elasticities" are not statistically different from the values of the factor shares. This has been illustrated by a consideration of Oulton and O'Mahony's two tests using panel data of UK manufacturing industries.

The argument is not affected if factor shares vary over time. All that one needs is a more flexible functional form (such as translog) to give a good fit to the data. These conclusions are the result of logic and not of subjective interpretation and it is surprising to see the continued uncritical widespread use of the aggregate production function in both empirical and theoretical studies.

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Table 1 **UK Total Industry. Selected Macroeconomic Variables for 1990 in current prices**

Value added (V)	£ 519,089 million
Rate of profit (r)	0.0988
Capital Stock (J)	£1,540, 000 million
Wage rate (w)	£13,017.72
Total persons employed (L)	28,189 million
Capital-output ratio (J/V)	2.9667
Capital's share (a)	0.2931
Labour's share(1-a)	0.7069
a^{-a}	1.4329
$(1-a)^{-(1-a)}$	1.2779

The Two Accounting Identities

(i) $V \equiv rJ + wL$

£519,089 million ≡ (0.0988)x(£1,540,000 million)+ (£13,017.72)x(28,189 million)

(ii) $V \equiv [\, a^{-a}(1-a)^{-(1-a)} r^{a} w^{(1-a)} \,] J^{a} L^{(1-a)} = AK^{a} L^{(1-a)}$

£519,089 million ≡ (1.43) x(1.28)x(0.51) x(£810.34)x(£3,731.35)x(184,774.58)

Sources: OECD Database, *Flows and Stocks of Fixed Capital, 1971-1996*, OECD, authors' estimates.

Table 2 Estimating Various Specifications of the Identity; Dependent Variable *MFPG* (equations (i) and (ii)) and Output Growth (equation (iii)), pooled sub-periods, 1954-1986

	MFPG		$\hat{Y}$
	(i)	(ii)[a]	(iii)[a]
$\hat{Y}_i$	0.817 (55.12)	-	-
$\hat{J}_i$	-0.095 (-6.53)	-0.015 (-0.33)	0.153 (3.13)
$\hat{L}_i$	-0.202 (-20.85)	0.061 (1.95)	0.311 (9.27)
$\hat{M}$	-0.493 (-37.49)	-0.040 (-1.82)	0.558 (23.70)
R^2	0.751	0.145	0.790

Notes: t-statistics in parentheses. Regressions (ii) and (iii) include a constant. [a] Fixed-effects estimation, time and industry dummies.
Source: Data from Oulton and O'Mahony (1994).
Memorandum item: Shares of inputs in gross output (figures in parentheses are the standard deviations); capital 0.141 (5.5); labour, 25.9 (8.2) and intermediate inputs, 60.0 (8.5).

Table 3 Estimating Various Specifications of the Identity: Dependant Variable $(\hat{Y}_{it} - \hat{L}_{it})$, pooled sub-periods 1954-1986.

	(i)	(ii)[a]
MFPG	0.913 (55.12)	-
$(\hat{J}_{it} - \hat{L}_{it})$	0.145 (9.68)	0.153 (4.80)
$\hat{L}_{it}$	0.001 (-0.05)	0.022 (0.52)
$(\hat{M}_{it} - \hat{L}_{it})$	0.597 (56.53)	0.558 (23.70)
R^2	0.925	0.503

Notes: t-statistics in parentheses. Equation (ii) includes a constant. [a]Fixed-effects estimation, time and industry dummies.
Source: Data from Oulton and O'Mahony (1994).