

The conditional convergence properties of simple Kaldorian growth models

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* The author is grateful to John McCombie for his helpful comments upon an earlier version of this paper. All remaining errors are the sole responsibility of the author.

Abstract: Findings of conditional convergence are usually interpreted within a neoclassical growth framework. This follows from the methodology of testing for conditional convergence, whereby the estimating equation is explicitly derived from a neoclassical growth model. Given this explicit derivation, findings of conditional convergence might be thought to discriminate against alternative approaches to growth in general and the Kaldorian approach to growth in particular. This paper shows, however, that this is not the case. It does so by examining the conditional convergence properties of the "core" model of Kaldorian growth theory- the Kaldor-Dixon-Thirlwall (KDT) model. In particular, it demonstrates that this model predicts conditional convergence of a qualitatively identical nature to that predicted by the neoclassical growth model. A simple extension of the KDT model that is reconciled with quantitative estimates of the speed of conditional convergence is also presented.

Keywords: growth, convergence, Kaldor.

JEL codes: E12, O40, C20

1. Introduction

Within the empirical growth literature, there exists a large and influential body of work documenting the existence of conditional convergence- the tendency for initially poorer economies to grow systematically faster than initially richer ones once a vector of other factors has been controlled for. In particular, conditional convergence has typically been found to exist regardless of the sample considered. Thus, not only has it been found for a broad sample of over 100 countries, but also, for example, for US states, European NUTS regions and Japanese prefectures (see, *inter alia*, Barro & Sala-i-Martin, 2004, and Sala-i-Martin, 1996).¹

Such findings of conditional convergence are usually interpreted within a neoclassical framework. Indeed, following the influential work of Barro & Sala-i-Martin (1991, 1992, 2004)² and Mankiw *et al* (1992), the conditional convergence properties of neoclassical growth models are well-known. Furthermore, their demonstration that one can derive from such models an explicit estimating equation for testing for conditional convergence (see, for example, BS, 2004, pp 111-113; Mankiw *et al*, 1992, pp 422-423), means that it is a neoclassical growth model that typically provides both the point of departure and theoretical underpinning for a study of conditional convergence. This is not only the case for studies that follow BS and Mankiw *et al* in employing a simple cross-sectional approach to testing for convergence (for a recent example see Henley, 2005), but also for studies that adopt a more sophisticated panel data approach designed to control for omitted heterogeneity (Caselli *et al*, 1996; Islam, 1995). In the former case, the rate of conditional convergence is, regardless of the sample used, typically estimated to lie in the range 1-3% per annum (Sala-i-Martin, 1996), whilst, in the latter, much higher estimates, more in the range 6%-30% per annum, are found (Canova & Marcet, 1995; Caselli *et al*, 1996; Islam, 1995). Nevertheless, to reiterate, the basic neoclassical framework of interpretation remains the same. Indeed, the only relevant question is taken to be- which variant of the neoclassical growth model is the estimated speed of conditional convergence most consistent with?

However, although dominant, the neoclassical approach to growth is not the only one that exists in macroeconomics. Moreover, amongst alternative approaches, one that has proved to be particularly influential amongst more heterodox inclined economists is, what may be termed, the Kaldorian approach to growth.³ In contrast to the supply-side emphasis of the neoclassical approach, this approach sees the growth process as involving a complex interplay between the supply- and demand-sides of an economy with demand often taking the lead. In particular, demand is often able to take the lead because, within limits, it is able to elicit a response from the supply-side of an economy in the form of induced technical progress, which results from the

¹ NUTS is an acronym for Nomenclature of Units of Territorial Statistics and represents a classification system for regions of the European Union (EU). This system was established by the EU's statistical agency- *Eurostat*.

² Henceforth, Barro & Sala-i-Martin will be referred to as BS.

³ This approach originates from the work of Nicolas Kaldor (see, *inter alia*, Kaldor, 1966, 1970, 1977, 1981). It has since been further developed by Cornwall (1977), Dixon & Thirlwall (1975), McCombie & Thirlwall (1994, 1997a, 1997b) and Setterfield (1997a, 1997b), to name but a few.

exploitation of dynamic economies of scale, and an elastic reaction from labour. Consequently, and, again, in contrast to the neoclassical approach, the natural rate of growth is characterised as being endogenous to demand growth, this being a fundamental proposition of the Kaldorian approach (León-Ledesma & Thirlwall, 2002; Thirlwall, 2003).⁴

Prima facie, this alternative, Kaldorian, approach to growth is very attractive⁵ and various authors have constructed formal models that capture its most important aspects. Most notable, perhaps, amongst these models is the Kaldor-Dixon-Thirlwall (KDT) model of Dixon & Thirlwall (1975). This is the standard model of Kaldorian growth theory in the sense that it provides the base upon which many other models in the field build (McCombie, 2002; Roberts, 2002).⁶ The question arises, however, as to the relationship between simple Kaldorian models such as the KDT model and the evidence on conditional convergence. This is especially so given the explicitly neoclassical framework within which convergence studies have been conducted and which has been highlighted above. It is this question of the conditional convergence properties of simple Kaldorian growth models in general, and the KDT model in particular, that this paper seeks to address.

The structure of the remainder of the paper is as follows. Section 2 provides an overview of the conditional convergence literature, emphasising its neoclassical foundations. Following this, section 3 introduces the KDT model and examines its conditional convergence properties. In particular, it proves the qualitative equivalence of this model with neoclassical growth theory with respect to the prediction of conditional convergence. It furthermore proves that an estimating equation for such convergence can be derived from the KDT model that is identical to that which BS, Mankiw *et al* and others derive from the neoclassical growth model. Section 4 then presents an extension of the KDT model that serves to reconcile it with quantitative estimates of the speed of conditional convergence found in panel data studies. Finally, section 5 concludes.

2. Conditional convergence and its neoclassical interpretation

The modern literature on convergence dates back to Baumol (1986), who, for a sample of 16 industrialised countries, found convergence over the long time period of 1870-1979. However, in broader samples of countries, no such convergence exists (De Long, 1988; Romer, 1986). Initially, this was taken as evidence against the

⁴ There are also differences between the neoclassical and Kaldorian approaches to growth at a methodological level. Thus, the former adopts an axiomatic-deductive approach to modelling, whilst the latter stresses the importance of realistic, or, what Kaldor (1972), termed, "scientific", assumptions in modelling.

⁵ Indeed, some of its central themes, such as the endogeneity of technical progress, have been incorporated into the neoclassical approach by "new" growth theorists such as Aghion & Howitt (1998) and Romer (1986, 1990).

⁶ Examples are De Benedictis (1998), Fingleton (2000), León-Ledesma (2002), Roberts (2002), Swales (1983), and Targetti & Foti (1996).

traditional neoclassical model of growth, the Solow-Swan model⁷, and provided the impetus for the development of new neoclassical growth models that allow for endogenous technical progress (Romer, 1986, 1990). This interpretation of the convergence evidence changed, however, with the introduction of the distinction between absolute and conditional convergence by BS (1991, 1992, 2004) and Mankiw *et al* (1992). Absolute convergence is the type of convergence that Baumol found—convergence that tends to result in the per capita incomes of poorer economies "catching-up" with those of richer economies. Meanwhile, conditional convergence corresponds to a tendency for initially poorer economies to grow systematically faster than initially richer economies *once a vector of other growth influences has been controlled for*. Although broad samples of countries lack absolute convergence, they do exhibit conditional convergence (BS, 2004, pp 521-523; Mankiw *et al*, 1992, pp 426, tables IV and V). This is significant because both BS (see, for example, 2004, pp 46-50) and Mankiw *et al* (1992, p 422) argue that, in general, the neoclassical growth model predicts conditional rather than absolute convergence. Moreover, BS and Mankiw *et al* explicitly derive their estimating equation for testing for conditional convergence from the neoclassical growth model, and this is now the standard approach to testing for convergence in the empirical growth literature.⁸ Consequently, in the literature, tests of conditional convergence are presented as if they are tests of that framework (see, *inter alia*, Henley, 2005; Islam, 1995; and Yao & Weeks, 2000). Although this approach has not gone without criticism from, for example, Quah (1993, 1996), the result has been a renaissance of the traditional neoclassical growth model, albeit in a number of slightly modified forms, the most notable of which is Mankiw *et al*'s (1992) Solow-Swan model augmented with human capital.⁹

To elaborate upon this neoclassical framework in which studies of convergence are framed, BS (see, *inter alia*, 2004) and Mankiw *et al* (1992) explicitly demonstrate that the following equation can be derived from the traditional neoclassical growth model:

$$r_{t,t+T} = \left(\frac{1 - e^{-\beta T}}{T} \right) \ln(A_t) + r_{t+T}^* + \left(\frac{1 - e^{-\beta T}}{T} \right) \ln\left(\frac{R}{A}\right)^* - \left(\frac{1 - e^{-\beta T}}{T} \right) \ln(R_t) \quad (1)$$

where $r_{t,t+T}$ denotes the average rate of labour productivity growth between periods t and $t + T$, and r_{t+T}^* the equilibrium rate of labour productivity growth. Most authors, including BS and Mankiw *et al*, take r_{t+T}^* to be uniform across economies, justifying this with the argument that technology is a pure public good. Meanwhile, R_t denotes the initial *level* of labour productivity, A_t the initial level of technology/efficiency and $(R/A)^*$ the steady-state level of output per effective worker. Finally, β is the speed of

⁷ More generally, it was taken as evidence against neoclassical exogenous growth models, including not only the Solow-Swan model, but also, for example, the Ramsey model as developed by Cass (1965) and Koopmans (1965).

⁸ Sala-i-Martin (1996) terms this the "classical approach" to testing for convergence.

⁹ Both absolute and conditional convergence are types of β convergence. Following BS (see, for example, BS, 2004, chapter 11), the convergence literature makes a further distinction between this type of convergence and σ convergence, where the latter is defined as a declining cross-sectional dispersion of levels of per capita (or per worker) income or product.

conditional convergence and varies with the version of the neoclassical growth model being considered (BS, 2004).

From equation (1), it can be seen that, *conditional* upon controlling for non-random variations in $\ln(A_t)$, r_{t+T}^* and $\ln(R/A)^*$, the neoclassical growth model predicts that the lower is an economy's initial level of labour productivity, the faster will be its subsequent rate of growth. Furthermore, equation (1) suggests the following estimating equation to test for conditional convergence:

$$r_{t,t+T} = a_0 + a_1 \ln(R_t) + \varphi \underset{\sim}{X}_t + u_{t,t+T} \quad (2)$$

where $\underset{\sim}{X}_t$ is a row vector of variables intended to capture all non-random variations in the variables other than $\ln(R_t)$ that appear in equation (1), φ is a corresponding column vector of coefficients and $u_{t,t+T}$ is an independently distributed disturbance term with mean zero and variance σ_u^2 .¹⁰ When BS and Mankiw *et al* estimate this equation using hybrid cross-sectional data they find $a_1 < 0$ to be significant, thereby confirming the prediction of conditional convergence. Moreover, from \hat{a}_1 they derive an explicit estimate of β that suggests a rate of conditional convergence in the range 1.3-2.9 % per annum depending upon the precise sample used (see Mankiw *et al*, 1992, p. 429, table VI; Sala-i-Martin, 1996, p. 1024, table 4). This is consistent with a version of the Solow-Swan model that has been augmented to include human capital (Mankiw *et al*, 1992).

However, within the convergence literature, both BS and Mankiw *et al* have come under heavy criticism. This is because in the vector of variables $\underset{\sim}{X}_t$ they fail to explicitly control for non-random variations in $\ln(A_t)$.¹¹ Yet, in reality, different countries do seem to exhibit systematic differences in levels of technology/efficiency that help to explain cross-country differences in labour productivity. This leads to the expectation of a positive correlation between $\ln(A_t)$ and $\ln(R_t)$ in equation (1), thereby creating an upward bias in BS and Mankiw *et al*'s estimates of \hat{a}_1 (see, *inter alia*, Canova & Marcet, 1995; Caselli *et al*, 1996; Islam, 1995; Temple 1999). This, in

¹⁰ BS (2004, pp 464-465) point out that the assumption of an independently distributed disturbance term will be violated if there exist shocks that have a common influence on sub-groups of economies. They view this possibility as being more likely when the economies in question are regions belonging to a single country or a common currency zone and, in such cases, supplement equation (2) with a variable to control for the influence of such shocks.

¹¹ This is a slightly unfair representation of BS (2004), if not Mankiw *et al* (1992). This is because BS do allow different countries to exhibit different levels of efficiency by adopting a broad interpretation of A_t which sees it as capturing not only the level of technology, but various kinds of country specific government policies, distortions and so forth. They attempt to control for these factors by including proxies for them in $\underset{\sim}{X}_t$. However, it is unlikely that the proxies capture all of the unobserved variations in A_t and, to the extent that this is the case, the problem of failing to control for non-random variations in $\ln(A_t)$ remains.

turn, implies a downward bias in their estimates of β . To overcome this problem, the literature has, following Islam (1995), adopted panel data techniques, allowing unobserved variations in technology/efficiency to be treated as economy-specific fixed effects. The result has been much higher estimates of the speed of conditional convergence- more in the range 6-30 % per annum (see, *inter alia*, Canova and Marcet, 1995; Caselli *et al*, 1996; Islam, 1995).¹² However, although these higher estimates of β have raised questions over Mankiw *et al*'s augmented Solow-Swan model, there has been no question that the correct model of growth is still the neoclassical model. Furthermore, the basic neoclassical framing of the convergence question, with the estimating equation being based on equation (1), has remained the same.

3. The KDT Model and Its Conditional Convergence Properties

3.1. Overview of the KDT model

As the standard model of Kaldorian growth theory, the KDT model, developed by Dixon & Thirlwall (1975)¹³, consists of four structural relationships:

$$y_t = \gamma x_t \quad (3)$$

$$x_t = -\eta\pi_{t-1} + \delta\pi_c + \varepsilon y_c \quad (4)$$

$$\pi_t = w + \tau - r_t \quad (5)$$

$$r_t = r_e + \lambda y_t \quad (6)$$

Equation (3) is an export-base relationship in which the growth rate of real output, y , is a positive linear function of the growth rate of real demand for exports, x . In turn, equation (4) models x as dependent upon, first, changes in the price competitiveness of the home economy relative to competitor economies and, second, the rate of growth of real income in markets that are being exported to, y_c . Changes in relative price competitiveness are captured by the negative impact of domestic price inflation, π , and the positive impact of competitor economy price inflation, π_c , on x . The magnitude of the impact of y_c on x depends upon the (foreign) income elasticity of demand for exports, ε , which is determined by the non-price competitiveness of domestic exports (McCombie & Thirlwall, 1994, 1997a, 1997b). Consistent with an imperfectly competitive export sector that practices mark-up pricing, equation (5) specifies that π is increasing in both the rate of nominal wage inflation, w , and the rate of mark-up growth, τ , in the domestic economy and decreasing in the rate of labour productivity growth, r .¹⁴ Finally, equation (6) is Verdoorn's law and specifies the

¹² Estimates towards the upper end of this range, such as those obtained by Canova and Marcet (1995), come from regional data. Islam (1995), meanwhile, obtains country estimates at the very bottom of this range. Caselli *et al* (1996) argue, however, that this is because of simultaneity problems in growth regressions that Islam (1995) fails to control for.

¹³ Dixon & Thirlwall developed the model as an explicit formalisation of a growth model first outlined by Kaldor (1970).

¹⁴ Both w and τ are assumed to be exogenously determined. For an extension of the KDT model that endogenises w see Roberts (2002).

existence of a long-run positive relationship between r and y . This relationship results from the assumed presence of economies of scale, and, in particular, dynamic economies of scale (McCombie & Thirlwall, 1994, pp 173-175), with λ denoting the so-called Verdoorn coefficient and r_e capturing exogenous determinants of labour productivity growth.

In the specification of the KDT model it has been assumed that changes in π affect x with a one-period lag.¹⁵ Crucially, this lag is responsible for the KDT model's transitional dynamics and, as shown below, it is these transitional dynamics that are responsible for the model's conditional convergence properties. It is possible to justify the assumption of a lag by the existence of the sort of recognition and order-delivery lags in exporting that Hooper & Marquez (1995), for example, identify. It can also be justified on the grounds that the reaction to changes in relative price competitiveness typically involves discrete shifts to new suppliers. However, because they incur adjustment costs, such shifts occur only gradually (Carlin & Soskice, 1990; Landesmann & Snell, 1989). Meanwhile, providing that one period of logical time is treated as equivalent to one calendar year, the assumption that the lag is one period in length is reasonably consistent with empirical work (see, in particular, Krugman, 1989; Landesmann & Snell, 1989). However, whilst, following Dixon & Thirlwall (1975), it has been assumed that the lag appears in the dynamic export demand function, it is worth remarking that the quantitative properties of the model, including the results derived below, are robust to instead specifying the lag as occurring in one of the three other relationships that make-up the model.

Given the one-period lag, it is straightforward to show that equations (3)-(6) reduce to:

$$r_t = \{r_e + \gamma\lambda[\delta\pi_c - \eta(w + \tau)]\} + \gamma\eta\lambda(r_{t-1}) \quad (7)$$

Assuming that $0 < \gamma\eta\lambda < 1$, it follows that the KDT model possesses a stable equilibrium solution in which r_t and y_t are given by:

$$r^* = \frac{r_e + \gamma\lambda[\delta\pi_c - \eta(w + \tau) + \varepsilon y_c]}{1 - \gamma\eta\lambda} \quad (8)$$

$$y^* = \frac{\gamma[\delta\pi_c - \eta(w + \tau - r_e) + \varepsilon y_c]}{1 - \gamma\eta\lambda} \quad (9)$$

These equations imply that r^* and y^* are both increasing in r_e , γ , λ , δ , π_c , ε and y_c and decreasing in w and τ , whilst the effect of a change in η is ambiguous.

3.2. *The conditional convergence properties of the KDT model*

As discussed, findings of conditional convergence are, following BS (1991, 1992, 2004) and Mankiw *et al* (1992), interpreted within a neoclassical framework and, indeed, the whole empirical methodology of testing for such convergence is derived

¹⁵ Strictly speaking, the lag should also apply to the variables π_c and y_c in equation (4). Their omission, however, does not affect any of the results derived below.

from the neoclassical growth model. From this, it might be thought that findings of conditional convergence discriminate against alternative approaches to growth in general and the Kaldorian approach to growth in particular, especially given the latter's focus on the importance of increasing returns. Thus, this is the position that Fingleton & McCombie (1998, pp 97-101) seem to adopt in discussing ‘The Keynesian [i.e. Kaldorian] versus neoclassical explanations of European economic growth.’ However, despite the assumed existence of increasing returns, simple Kaldorian growth models, including the KDT model, can be shown to predict conditional convergence that is qualitatively identical to that generated by the traditional neoclassical growth model. In particular, for the KDT model, it is possible to state the following proposition:

Proposition 1: the KDT model predicts conditional convergence of a form that is qualitatively identical to that predicted by the neoclassical growth model.

Proof: Simply manipulate equations (7) and (8) of the KDT model to give:

$$r_t - r_{t-1} = -\beta(r_{t-1} - r^*) \quad (10)$$

where $\beta = 1 - \gamma\eta\lambda$. Provided $0 < \gamma\eta\lambda < 1$, it follows that an economy whose initial growth rate is greater than its equilibrium growth rate, i.e. $r_{t-1} > r^*$, will experience a falling growth rate over time, i.e. $r_t - r_{t-1} < 0$. This will continue until $r_t = r^*$. Conversely, an economy whose initial growth rate is less than its equilibrium growth rate, i.e. $r_{t-1} < r^*$, will experience a growth rate that is increasing over time, i.e. $r_t - r_{t-1} > 0$. Again, this will continue until $r_t = r^*$. Such transitional dynamics are, however, identical to those of the neoclassical growth model. Given this, the KDT model predicts conditional convergence of a form that is qualitatively identical to that found in neoclassical growth theory. A major difference, however, is that whilst the traditional neoclassical growth model assumes that the equilibrium growth rate is determined by an exogenously given rate of technical progress and is therefore the same for all economies (see section 2), the KDT model allows the growth rate to vary across economies in accordance with variations in the parameters and exogenous variables that appear in equation (8).¹⁶ □

Furthermore, given proposition 1, it can be shown that an equation identical to equation (1) can be derived from the KDT model, thereby leading to proposition 2, which is stated below. This is significant because, as discussed, it is this equation that provides the theoretical underpinning for the neoclassical interpretation of findings of conditional convergence.

¹⁶ The importance of this difference cannot be understated. In particular, it is this difference that better permits the KDT model to explain *sustained* long-run differences in growth performance both between countries (and continents) in the world (for example, between Africa and South East Asia) and between regions within countries.

Proposition 2: from the KDT model it is possible to derive a conditional convergence equation identical to that which can be derived from the neoclassical growth model.

Proof: define an arbitrary variable, A_t , that is assumed to grow exogenously at the rate r^* , so that approximately:

$$\ln(A_t) = \ln(A_0) + r^* t \quad (11)$$

In effect, this defines the level of labour productivity in period t of an economy that has been in continuous equilibrium in the KDT model. Given this, further define:

$$\tilde{R}_t = \frac{R_t}{A_t} \quad (12)$$

Take (natural) logarithms of both sides of equation (12) and subtract $\ln(\tilde{R}_{t-1})$ from both sides:

$$\begin{aligned} \ln(\tilde{R}_t) - \ln(\tilde{R}_{t-1}) &= [\ln(R_t) - \ln(R_{t-1})] - [\ln(A_t) - \ln(A_{t-1})] \\ &= r_t - r^* \end{aligned}$$

Define $\tilde{r}_t = \ln(\tilde{R}_t) - \ln(\tilde{R}_{t-1})$ so that:

$$\tilde{r}_t = r_t - r^* \quad (13)$$

This implies that \tilde{r}_t will obey identical dynamics to r_t in equation (10).¹⁷

In turn, this implies that $\ln(\tilde{R}_t)$ will also obey identical dynamics:¹⁸

$$\ln(\tilde{R}_t) - \ln(\tilde{R}_{t-1}) = -\beta[\ln(\tilde{R}_{t-1}) - \ln(\tilde{R}^*)] \quad (14)$$

Add $\ln(\tilde{R}_{t-1})$ to both sides of equation (14) and iterate forward to obtain the general solution for $\ln(\tilde{R}_t)$:

$$\ln(\tilde{R}_t) = [1 - (1 - \beta)^t] \ln(\tilde{R}^*) + (1 - \beta)^t \ln(\tilde{R}_0) \quad (15)$$

¹⁷ This follows from substituting equation (13) into equation (10) so as to obtain $\tilde{r}_t - \tilde{r}_{t-1} = -\beta(\tilde{r}_{t-1} - \tilde{r}^*)$.

¹⁸ Thus, integrating $\tilde{r}_t - \tilde{r}_{t-1} = -\beta(\tilde{r}_{t-1} - \tilde{r}^*)$ with respect to time gives $\ln(\tilde{R}_t) - \ln(\tilde{R}_{t-1}) = -\beta[\ln(\tilde{R}_{t-1}) - \ln(\tilde{R}^*)] + K$. This corresponds to equation (14) except that the constant of integration has been set to zero. This is because $\ln(\tilde{R}_t) - \ln(\tilde{R}_{t-1}) = \ln(\tilde{R}_t) - \ln(\tilde{R}^*) = 0$ must hold in equilibrium.

It follows that the (natural) log difference in $R_{\sim t}$ between periods t and $t + T$ is:

$$\ln(R_{\sim t+T}) - \ln(R_{\sim t}) = [1 - (1 - \beta)^T] \ln(R_{\sim}^*) - [1 - (1 - \beta)^T] \ln(R_{\sim t}) \quad (16)$$

From equations (12) and (11):

$$\begin{aligned} \ln(R_{\sim t+T}) &= \ln(R_{\sim t+T}) - \ln(A_{t+T}) \\ &= \ln(R_{t+T}) - \ln(A_t) - r^* T \end{aligned} \quad (17)$$

Substitute equation (17) into equation (16), use $\ln(R_{\sim t}) = \ln(R_t) - \ln(A_t)$, re-arrange and divide by T :

$$r_{t,t+T} = \left[\frac{1 - (1 - \beta)^T}{T} \right] \ln(A_t) + r^* + \left[\frac{1 - (1 - \beta)^T}{T} \right] \ln\left(\frac{R}{A}\right)^* - \left[\frac{1 - (1 - \beta)^T}{T} \right] \ln(R_t) \quad (18)$$

Equation (18) is a discrete time version of equation (1). \square

Essentially, what the proof of proposition 2 does is define a variable, A_t , that plays an analogous role to that played by the level of technology/efficiency in the neoclassical growth model. It is the definition of this variable combined with the fact that the rate of labour productivity growth obeys qualitatively identical dynamics in the KDT model to the neoclassical growth model that enables the derivation of equation (18). In turn, given the equivalence of equation (18) with equation (1), it obviously follows that any finding of conditional convergence obtained by estimating equation (2) is (qualitatively) consistent not only with a neoclassical approach to growth, but also with a Kaldorian approach.

4. Conditional Convergence in a Modified KDT Model

From the above, it would seem that, when it comes to the prediction of conditional convergence, the KDT model is observationally equivalent to the neoclassical growth model. This is true, but it has been seen that, in the convergence literature, estimates of the speed of convergence have been used not only to help justify a neoclassical approach to growth in general, but also to discriminate between *different* variants of the neoclassical growth model. Thus, for example, BS and Mankiw *et al* have used their slow estimated rates of conditional convergence to argue that the Solow-Swan model should be augmented with human capital, whilst Caselli *et al* (1996) see the higher estimated rates obtained with panel data as being consistent with open economy versions of the Ramsey model. This begs two questions of the KDT model. First, what is the speed of conditional convergence predicted by the model? and, second, is this predicted speed consistent with those found in the convergence literature?

To answer the first question, note that, from the proofs of propositions 1 and 2, the theoretical expression for the speed of conditional convergence predicted by the KDT model is $\beta = 1 - \gamma\eta\lambda$. Given this expression, it is possible, by assigning benchmark values to γ , η and λ , to derive a quantitative value for the predicted speed. Thus, it seems reasonable to assume that $\gamma = 1$ because any other value would cause the share of exports in output to explode, whilst empirical literature indicates that benchmark values of $\eta = \lambda = 0.50$ are reasonable.¹⁹ Taken together, these values imply $\beta = 0.75$ or a predicted rate of conditional convergence of 75% per annum.²⁰

The fact that the KDT model predicts such a fast rate of conditional convergence provides an immediate answer to the second question. Somewhat ironically, the findings of the convergence literature are inconsistent with the KDT model, not because the model does not predict conditional convergence, but because it predicts conditional convergence that is *too fast*. To understand why this is so, it is necessary to realise that, *in any growth model*, the prediction of conditional convergence arises from its transitional dynamics. Providing a growth model possesses stable transitional dynamics, it will predict conditional convergence.²¹ Furthermore, the speed at which conditional convergence is predicted to occur depends upon the speed at which the dynamics in the model in question operate. The transitional dynamics of the KDT model are provided by its process of "circular and cumulative causation" which results from the positive feedback from output growth to labour productivity growth that equation (6) in the model provides. Therefore, again somewhat ironically, the reason why the model predicts a much higher rate of conditional convergence than has been found empirically is because the dynamics are *too stable* or, in other words, the mechanism of "circular and cumulative causation" built into the model is *too weak*.²²

Given the above, however, it follows that the KDT model can be reconciled with quantitative estimates of the speed of convergence if it can be modified so as to slow down its transitional dynamics and, thereby, strengthen its mechanism of circular and cumulative causation. A simple example of how this may be done may be provided

¹⁹ Hooper & Marquez (1995) survey a wide range of studies reporting point estimates of η for the UK, whilst Anderson (1993), Bairam (1988, 1993), Driver & Wren-Lewis (1999), and Landesmann & Snell (1989) also provide point estimates of η for the UK. The average point estimate of η from the studies surveyed by Hooper & Marquez and the other studies referred to is 0.43 (see Roberts, 1999). Meanwhile, surveys of the empirical literature (Bairam, 1987; McCombie & Thirlwall, 1994, chapter 2) on Verdoorn's law reveal a value of 0.50 to be reasonable.

²⁰ This result does, however, display a high degree of sensitivity to the assumed parameter values. Thus, in their original article setting out the KDT model, Dixon & Thirlwall (1975, p 212) argue for a value of $\eta = 1.5$. With this value of η , the predicted speed of conditional convergence becomes 25% per annum. This removes further need for extension of the model along the lines set out below.

²¹ This is why neoclassical endogenous growth models of the simple AK variety do not predict conditional convergence- they possess no explicit transitional dynamics.

²² By contrast, the Solow-Swan and augmented Solow-Swan models predict slow rates of conditional convergence because, compared to the KDT model, their transitional dynamics are relatively unstable.

by realising that Verdoorn's law in equation (6), which provides the linchpin of the KDT model's circular and cumulative growth mechanism, is intended as a *long-run* relationship. In particular, contrary to the implicit assumption in the specification of equations (3)-(6), it is unreasonable to assume that labour productivity growth is able to respond instantaneously to output growth through the mechanism of dynamic increasing returns. Indeed, Setterfield (1997a, p 367) argues that "the realization of induced technical progress through the Verdoorn Law may require the accumulation of specific new capital, which will only come into productive use, and so enhance productivity, in some future period." Furthermore, for the very reason that the law is properly conceived of as a long-run relationship, it is ordinarily estimated using cross-sectional growth rates over five to ten-year periods (McCombie & Roberts, 2006). Given this, it seems reasonable to rewrite the KDT model in the following manner:

$$y_t = \gamma x_t \quad (3)$$

$$x_t = -\eta \pi_{t-1} + \delta \pi_c + \varepsilon y_c \quad (4)$$

$$\pi_t = w + \tau - r_t \quad (5)$$

$$r_{LR} = r_e + \lambda y_t \quad (6')$$

$$r_t = r_{t-1} + \theta(r_{LR} - r_{t-1}) \quad (19)$$

where equation (6') replaces equation (6) in the original KDT model with the LR subscript denoting the fact that Verdoorn's law is now explicitly and properly recognised as a long-run relationship, whilst equation (19) has been added to capture the slow adjustment of actual labour productivity growth to its long-run level. Obviously, the exact speed of the adjustment is determined by the parameter θ , where $0 < \theta < 1$ is assumed.

In this modified KDT model, the steady-state solutions for labour productivity (r^*) and real output growth (y^*) remain the same as in the original KDT model, being given by equations (8) and (9) respectively. However, the difference equation which provides the underlying transitional dynamics becomes:

$$r_t = \theta\{r_e + \gamma\lambda[\delta\pi_c - \eta(w + \tau) + \varepsilon y_c]\} + [1 - \theta(1 - \gamma\eta\lambda)]r_{t-1} \quad (20)$$

which implies that r^* and y^* will be stable provided $-1 < 1 - \theta(1 - \gamma\eta\lambda) < 1$ and that the predicted rate of conditional convergence is given by $\beta' = \theta(1 - \gamma\eta\lambda)$ compared to $\beta = 1 - \gamma\eta\lambda$ in the original KDT model. Given that $0 < \theta < 1$, the predicted speed of conditional convergence is therefore necessarily less than in the original KDT model. Indeed, for plausible values of θ , this modified KDT model is easily capable of generating values of β consistent with those found in the panel data studies of conditional convergence that were discussed earlier.²³ Thus, the model will generate a predicted speed of conditional convergence of 6 % per annum if $\theta = 0.08$, whilst it will generate a predicted speed of 30 % per annum if $\theta = 0.40$. The former value of θ implies an adjustment half-life of r_t to r_{LR} of 8.7 years, whilst the latter value implies

²³ The simple Kaldorian models of growth investigated in this paper suggest panel data studies allowing for economy-specific fixed effects are to be preferred to simple cross-sectional studies because of persistent variations in r^* that might not otherwise be controlled for.

an adjustment half-life of 1.74 years. Both are compatible with the idea that it takes at least one business cycle to allow for the full realisation of dynamic economies of scale through Verdoorn's law.

5. Conclusion

In conclusion, this paper has examined the conditional convergence properties of simple Kaldorian growth models. Specifically, it has examined the conditional convergence properties of the KDT model, which is the standard model of Kaldorian growth theory, before progressing to a modified version of this model that allows for the slow realisation of dynamic economies of scale through Verdoorn's law. Whilst both models predict conditional convergence that is qualitatively identical to that predicted by the neoclassical growth model, only the latter predicts a speed of convergence that is quantitatively consistent with previous empirical work and, in particular, panel data studies of convergence. However, important to note is that the modification presented is by no means the only one that could plausibly reconcile the KDT model with quantitative estimates of the speed of convergence. Indeed, any modification which serves, in a plausible manner, to reduce the speed of the model's transitional dynamics and, therefore, strengthen its process of circular and cumulative causation will serve the same purpose.

The important lesson of the paper is that the neoclassical framework within which studies and findings of conditional convergence are usually presented is misleading. This is because the estimating equation that forms the theoretical basis for testing for such convergence can just as easily be derived from simple Kaldorian models of growth as it can from the neoclassical model of growth. Consequently, as a means of discriminating between growth paradigms, findings and estimates of the speed of conditional convergence serve no purpose. However, to finish, the analysis of this paper does suggest a possible way in which future empirical work using aggregative data might be able to discriminate between the neoclassical and Kaldorian approaches to growth. In particular, equations (8) and (18) suggest that the key distinguishing feature between the neoclassical growth model and Kaldorian growth models lies not in the prediction of conditional convergence, but in the prediction as to what that convergence *is conditional upon*. Thus, in contrast to the neoclassical growth model, convergence in the KDT model is predicted to be conditional upon, *inter alia*, cross-economy differences in the income elasticity of demand for exports that reflect differences in non-price competitiveness. It is therefore at the level of the potential conditioning variables, as well as the interpretation of these variables, that the substantive differences between the alternative growth paradigms lies. From this, it follows that the closer investigation of these variables in conditional convergence style regressions holds the potential to discriminate between the neoclassical and Kaldorian approaches to growth.

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