

ON ACCOUNTING IDENTITIES, SIMULATION EXPERIMENTS AND
AGGREGATE PRODUCTION FUNCTIONS: A CAUTIONARY TALE FOR
(NEOCLASSICAL) GROWTH THEORISTS.

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INTRODUCTION

A *sine qua non* of neoclassical growth theory is the existence of an aggregate production function. It is the very first equation of Solow's (1957) seminal paper. The widely used growth accounting approach, following Solow's (1957) seminal work, as well as the recent developments in endogenous growth theory, are grounded in the aggregate production function. (See, for example, Barro and Sala-i-Martin, 2004, especially chapters 4 and 10.) Yet it has been known for a long time just how flimsy are its theoretical foundations. Indeed, Solow (1957, p. 312) himself conceded that "it takes something more than the usual 'willing suspension of disbelief' to talk seriously of the aggregate production function". But this reservation was quickly glossed over – it "is only a little less legitimate a concept than, say, the aggregate consumption function".

The theoretical criticisms of the aggregate production function involve both the "aggregation problem" that dates from the 1940s and the Cambridge Capital Theory Controversies of the 1960s and 1970s. Fisher (1992) has shown with respect to the former that the problems of aggregation are so severe that the aggregate production cannot be said to exist – not even as an approximation.² The Cambridge Capital Theory Controversies proved to be more controversial and generated a great deal of heated debate in the leading academic journals. Fisher (2003) has argued that the issues involved are merely a subset of a more general aggregation problem, although Cohen and Harcourt (2003 a&b) consider that there is more to it than that. Nevertheless, whatever viewpoint one subscribes to, both serve to demonstrate the shortcomings of the neoclassical production function.

It is remarkable that although these arguments have been around for over half a century and while they were briefly acknowledged in textbooks and surveys in the 1970s, any reference to them has all but completely disappeared from the current literature. This is notwithstanding there has been no convincing refutation of these criticisms. They have simply been assumed away or ignored.

So why is the aggregate production function so widely and uncritically used? The answer seems to involve a form of Friedman's (1951) methodological instrumentalism. All theories, so the argument goes, involve heroic abstraction and unrealistic assumptions, but what matters is their predictive ability. The aggregate

² For a survey of these issues see Felipe and Fisher (2003).

production function, it is argued, passes this test with flying colours. The problem with this defence, as we shall show, is that the estimation of a putative aggregate production function using constant-price monetary (value) data cannot provide any inferences about the values of the putative parameters of the production function (output elasticities, aggregate elasticity of substitution) or the rate of technical progress. The reason is that there is an underlying accounting identity that relates these variables. This identity can be easily rewritten in a form that resembles a production function. This precludes any meaningful estimation of the “production function” and interpretation of the coefficients as estimates of an underlying technology. This critique is arguably the most damaging for the aggregate production function, because it applies even if there were no aggregation problems.

This is not a new critique, but first came to prominence in a rudimentary form in Phelps Brown’s (1957) criticism of Douglas’s cross-industry regression results (see, for example, Douglas, 1948), and elements of it can be traced back to Bronfenbrenner (1944) and Marshak and Andrews (1944). The critique was later formalised by Simon and Levy (1963) and Shaikh (1974, 1980, 1987) generalised it to time-series estimation of production functions. Simon (1979a) also considered the criticism in the context of both cross-section and time-series data and thought it serious enough to mention it in his Nobel prize lecture (Simon, 1979b). The criticism was re-examined and extended by Felipe and Adams (2005), Felipe and McCombie (2001, 2003, 2005 a&b, 2006, 2007), Felipe (2001 a&b), Felipe and Holz (2001), McCombie (1987, 1998 a & b, 2000-2001, 2001), McCombie and Dixon (1991) and McCombie and Thirlwall (1994). The critique as applied to cross-section data was also “rediscovered” by Samuelson (1979).

While Cramer (1969), Wallis (1973) and Intriligator (1978) in their econometric textbooks, and Walters (1966) in his survey on production and cost functions, have mentioned the argument, none pushed it to its logical conclusion: namely, that it invalidates any attempt to test, or estimate, the aggregate production function, *per se*. (See McCombie, 1998a, for a discussion.) Solow (1974, 1987), it is true, did attempt refutations of a couple of aspects of the critique, but these are not compelling (Shaikh, 1980, McCombie, 2001, Felipe and McCombie, 2005a).

The implications of the critique are far reaching. It implies that all those areas of neoclassical macroeconomics that use the aggregate production function (with, or without, the assumption that factors are paid their marginal products) have no theoretical or empirical basis. Because of the accounting identity, any estimation of a putative

aggregate production function can be made, through a suitable specification, to give a perfect fit to the data with constant returns to scale and with the output elasticities equalling the respective factor shares. This is true even though the aggregate production function does not exist and, for example, individual firms may be subject to substantial returns to scale. Consequently, the estimation of aggregate production functions is problematic, to say the least.

One way of forcefully illustrating the critique is to use simulation experiments. The advantage of this approach is that it allows us to know precisely what is the underlying micro-structure of the economy. Suppose, for example, the Cobb-Douglas production function gives a good fit to the aggregated data when we know that either the underlying technology of the firms in no way resembles the Cobb-Douglas production function, or, if it does, the conditions for successful aggregation are (deliberately) violated. This should at least give us reason to pause for thought. To this end, we review four simulation exercises that clearly demonstrate just how flimsy are the foundations of the aggregate production function and, hence, neoclassical growth theory. First, however, we briefly review the critique.

AGGREGATE PRODUCTION FUNCTIONS AND THE ACCOUNTING IDENTITY

The standard analysis of neoclassical production theory is well known and so is only briefly recapitulated here. The production function, which is essentially a microeconomic concept, in a general form is written as:

$$Q_t = f(K_t, L_t, t) \quad (1)$$

where Q , K , L , and t are output, capital, labour and a time trend that acts as a proxy for technical change. Theoretically, Q and K should be measured in *homogenous physical units* as equation (1) is a technological relationship (Ferguson, 1971). Equation (1) may be expressed in growth rates as:

$$\hat{Q}_t = \lambda_t + \alpha_t \hat{K}_t + \beta_t \hat{L}_t \quad (2)$$

The symbol $\hat{}$ above a variable denotes a growth rate. α and β are the technologically-determined output elasticities of capital and labour and λ is the rate of technical change, all of which may change over time.

If there is perfect competition and firms are paid their marginal products, then it can be simply shown that the following holds:

$$\hat{Q}_t = \lambda_t + a_t \hat{K}_t + (1 - a_t) \hat{L}_t \quad (3)$$

where a_t and $(1 - a_t)$ are the factor shares.

From Euler's theorem, using equation (1) output may be written in constant-price value terms as:

$$p_0 Q_t = p_0 f_{K_t} K_t + p_0 f_{L_t} L_t = \rho_t K_t + w_t L_t \quad (4)$$

where ρ is the rental price of each machine (i.e. the price per unit of time) and w is the wage rate, both measured in constant-price money terms and p_0 is the base-year price. From the dual, given the usual neoclassical assumptions, equation (3) can be derived by differentiating equation (4) as:

$$\hat{Q}_t = a_t \hat{\rho}_t + (1 - a_t) \hat{w}_t + a_t \hat{K}_t + (1 - a_t) \hat{L}_t \quad (5)$$

where $\lambda_t = a_t \hat{\rho}_t + (1 - a_t) \hat{w}_t$.

Such a discussion appears in all standard microeconomic textbooks and is carried seamlessly over into macroeconomic textbooks with no discussion of the problems involved in applying this analysis to the whole economy or a particular industry.

But, as we noted above, constant-price monetary data have to be used empirically to measure both output and capital, and it is here that an insurmountable difficulty arises both at the firm and industry levels. From the national accounts, the following identity must always hold at any level of aggregation:³

$$V_t \equiv r_t J_t + w_t L_t \quad (6)$$

where r is the rate of profit (a pure number) and w is the average real wage rate. V is value added and J is the constant price value of the capital stock, usually calculated by using the perpetual inventory method. We use V instead of Q and J instead of K to

³ The argument equally applies to gross output, when materials are included as an input.

emphasise the distinction between constant-price monetary values and physical units. The total compensation of capital is given by the rate of profit (which in competitive capital markets equals the rate of interest) multiplied by the constant price value of the capital stock, i.e., $r_t J_t$. It also equals the rental price of capital multiplied by the number of machines i.e., $\rho_t K_t$. Consequently, relationship between J_t and K_t is $J_t = (\rho_t/r_t)K_t$.⁴ In other words, from equation (6), the sum of total profits and the total compensation of labour *must* equal value added. Equation (6) can also be written, in growth rates, as:

$$\hat{V}_t \equiv a_t \hat{r}_t + (1 - a_t) \hat{w}_t + a_t \hat{J}_t + (1 - a_t) \hat{L}_t \quad (7)$$

It can readily be seen that equation (7) is formally equivalent to equation (5) when the latter is summed over firms and Q and K are expressed in constant prices.⁵ In these circumstances, \hat{r}_t , which is the growth of the rate of profit (a pure number), equals $\hat{\rho}_t$. But it should be noted that equation (7) does not require *any* of the neoclassical assumptions used to derive equation (5), including the existence of an aggregate production function. Thus, equation (5), when expressed using monetary values for output and capital, must always hold by virtue of the identity given by equation (6), and may give the misleading impression that equation (5) holds for any level of the economy, notwithstanding the aggregation problems which are erroneously assumed to be negligible.

Neoclassical production theory generally uses a specific functional form for equation (1), such as a Cobb-Douglas, CES, or translog production function. This is then estimated to derive values for the parameters of interest, such as the aggregate elasticity of substitution. This does not affect the argument. If equation (6) is expressed in *instantaneous* growth rates and then integrated, we derive purely as a result of a mathematical transformation, the result that at a specific time τ :

$$V_\tau \equiv r_\tau J_\tau + w_\tau L_\tau \quad (8)$$

$$\equiv B_o r_\tau^{a_\tau} w_\tau^{(1-a_\tau)} J_\tau^{a_\tau} L_\tau^{(1-a_\tau)} \quad (9)$$

$$\equiv A_\tau J_\tau^{a_\tau} L_\tau^{(1-a_\tau)} \quad (10)$$

⁴ For expositional ease we ignore capital gains/loses and obsolescence.

⁵ We ignore the aggregation problems.

B is the constant of integration and is equal to $a_\tau^{-a_\tau} (1 - a_\tau)^{-(1-a_\tau)}$. The shares are ‘constant’ because only one point of time (τ) is being considered. Consequently, if we use data for an economy or industry for, say, any one year, then the right-hand-side of equations (8), (9) and (10) will give *identical* values for value added. Consequently, at any point of time, a Cobb-Douglas will always give a good fit to the data, simply as an alternative mathematical way of writing the identity given by equation (6). More generally, if several periods are considered, equation (10) is an alternative way of writing the accounting identity if factor shares are constant over the time periods being considered.

If we use cross-industry or cross-regional data and estimate $V_i = AJ_i^\alpha L_i^\beta$ in logarithmic form, it follows from equation (10) that we should find an almost perfect fit to the extent that the variation in the logarithm of the wage rate and the rate of profit is small and the factor shares do not greatly differ between observations. This is precisely what Douglas’s many cross-sectional regressions in the 1930s found, with the coefficients on capital and labour nearly identical to their factor shares. Although, of course, this result is purely an artefact of the accounting identity, Douglas (erroneously) concluded that it proved the neoclassical theory of distribution and refuted the Marxian theory (Douglas, 1976).

Returning to time-series estimation, a stylised fact is that there is no discernible trend in the rate of profit, i.e., $\hat{r}_t = 0$, over the long run and the growth of the real wage grows at roughly a constant rate, i.e., $\hat{w}_t = \hat{w}$. Moreover, it is generally found that factor shares are roughly constant over time, i.e. $a_t = a$ and $1 - a_t = 1 - a$. (A constant mark-up pricing policy will, *inter alia*, give this result.)⁶ Hence the identity given by equation (6) may be expressed as:

$$V_t \equiv r_t J_t + w_t L_t \equiv A_0 e^{\lambda t} J_t^a L_t^{(1-a)} \quad (11)$$

where $\lambda = (1 - a)\hat{w}$. Equation (11) is nothing more than the accounting identity, but resembles a Cobb-Douglas relationship where $\alpha \equiv a$ and $(1 - \alpha) \equiv (1 - a)$.

But why do estimations of production functions not always give good statistical fits? The fact that they do not may give the impression that production functions are

⁶ Fisher (1971) showed using simulation analysis that constant aggregate factor shares are not the result of an aggregate Cobb-Douglas production function. See the discussion of Fisher’s simulation in the next section.

actually behavioural equations. The poor regression results could be due to two reasons. First, factor shares may vary considerably over the estimation period and, secondly, the path over time of the weighted rate of profit and the wage rate ($a_t \hat{r}_t + (1 - a_t) \hat{w}_t$) may not be sufficiently accurately proxied by a linear time-trend (λ). In other words, the two assumptions to transform equation (6) into equation (11) may be empirically incorrect. Using simulation analyses, McCombie (1998) and Felipe and Holz (2001) have shown that variations in factor shares do not prevent the Cobb-Douglas form from generally yielding acceptable results.

It is the second assumption, that is, the approximation of ($a_t \hat{r}_t + (1 - a_t) \hat{w}_t$) through a linear trend that is more often incorrect, and this can significantly bias the coefficients on the capital and labour variables and can even be responsible for suggesting, for example, that there are increasing returns to scale. But the fit to the identity can always be improved by the introduction of a suitable non-linear time trend (and there is nothing in neoclassical production theory that says technical progress has to be a linear function of time). Alternatively, including a suitable capacity utilisation variable or adjusting the capital and labour input for the intensity of use can have the same effect. If factor shares vary over time, then a functional form that is more flexible than the Cobb-Douglas (such as a Box-Cox transformation, which turns out to be similar to the CES) could always be used. This implies that if the path of the factor shares is not assumed to be constant, equation (6) can be transformed into functional forms that resemble CES or translog production functions. See, for example, Felipe and McCombie (2001) for the derivation of the CES from the identity.

The argument is simple and devastating. There is no point in estimating production functions using value (monetary) data. There are qualifications, such as the difference between the *ex post* rate of profit used in the identity and the neoclassical concept of the rental price of capital, but this does not significantly affect the argument and will not be considered here (see Felipe and McCombie, 2007).

The argument for the Cobb-Douglas production function is summarised in Table 1 where it is assumed that constant factor shares result from a constant mark-up pricing policy (although there are other reasons why factor shares do not show much variation over time). We next turn to a consideration of four simulation exercises that illustrate the issues involved.

Table 1 The Relationship Between the Accounting Identity and the Aggregate Cobb–Douglas Production Function Using Time-Series Data

The Accounting Identity

Prices are a mark-up on unit labour costs for firm i :

$$p_i = (1 + \pi_i) \frac{w_i L_i}{Q_i}$$

A constant mark-up gives constant shares of capital (a) and labour ($1-a$) in total value added, regardless of the underlying technology.

$$a = \pi/(1+\pi) \text{ and } (1-a) = 1/(1+\pi)$$

The accounting identity is given by:

$$p_i Q_i \equiv V_i \equiv r_i J_i + w_i L_i$$

$$\text{where } r_i = (p_i Q_i - w_i L_i)/J_i$$

Summing over industries gives:

$$V = \sum_i p_i Q_i = rJ + wL$$

There are no serious aggregation problems. Aggregation may actually reduce the variability of the aggregate factor share compared with the individual factor shares.

By definition (and making *no* assumption about the state of competition or the mechanism by which factors are rewarded) the following conditions hold:

$$\frac{\partial V}{\partial J} \equiv r \text{ and } \frac{\partial V}{\partial L} \equiv w$$

Given constant factor shares, the accounting identity at time t may be written as:

$$V_t = B r_t^a w_t^{(1-a)} J_t^a L_t^{(1-a)}$$

or, assuming the stylized fact that $a \hat{r}_t + (1-a) \hat{w}_t = (1-a) \hat{w} = \lambda$, as:

$$V_t = B e^{\lambda t} J_t^a L_t^{(1-a)}$$

Estimating $\ln V_t = c + b_1 t + b_2 \ln J_t + b_3 \ln L_t$ gives estimates of b_2 and b_3 exactly equal to the factor shares for *definitional* reasons: i.e. $b_2 = a$, and $b_3 = (1-a)$.

It is always possible to find an approximation that will give a perfect statistical fit to the data.

The Neoclassical Production Function

The micro production function with constant returns to scale is given by:

$$Q_i = A_0 e^{\lambda t} K_i^\alpha L_i^{(1-\alpha)}$$

Aggregation problems and the Cambridge Capital Theory Controversies show that theoretically the aggregate production function does not exist. Nevertheless, it is assumed that:

$$\sum_i Q_i = Q = A_0 e^{\lambda t} K^\alpha L^{(1-\alpha)}$$

Assuming (i) perfect competition and the (ii) aggregate marginal productivity theory of factor pricing gives:

$$p \frac{\partial Q}{\partial K} = p f_K = \rho \text{ and } p \frac{\partial Q}{\partial L} = p f_L = w$$

From Euler's theorem:

$$Q = f_K K + f_L L$$

and the cost identity is:

$$pQ = \rho K + wL \text{ or } Q = (\rho/p)K + (w/p)L$$

where ρ/p and w/p are physical measures and equal f_K and f_L . It is assumed for empirical analysis that $Q = V$ and $(\rho/r)K = J$ where r is the rate of interest, which is assumed to equal the rate of profit. Hence, $\rho K = rJ$.

Using time-series data and estimating $\ln V_t = c + b_1 t + b_2 \ln J_t + b_3 \ln L_t$ provides estimates of b_2 and b_3 , which are the aggregate output elasticities of labour and capital. If a good statistical fit is found, it is inferred that the estimation has not refuted the hypothesis of the existence of the aggregate production function.

The estimates of b_2 and b_3 equal the observed factor shares, i.e.,

$$b_2 = \alpha = a \text{ and } b_3 = (1-\alpha) = (1-a)$$

if assumptions (i) and (ii) above hold. If this is found to occur, it constitutes a failure to refute the theory that markets are competitive and factors are paid their marginal products.

Table 2 (cont.) *The Equifinality Theorem* (Felipe and McCombie, 2003).

Estimating $\ln V = c + b_1 t + b_2 \ln J + b_3 \ln L$ will always give a perfect fit to the data, provided factor shares are constant and the stylized fact $a \hat{r}_t + (1-a) \hat{w}_t = (1-a) \hat{w} = \lambda$ holds. This is the case irrespective of whether there is a “true” underlying aggregate Cobb-Douglas production function (no matter how theoretically implausible this may be) or no aggregate production function exists at all. The data cannot discriminate between these two cases. (The same result holds using growth rates.) If the condition of constant factor shares and a constant growth of the weighted wage and profit rates is not met, it is still possible to obtain a perfect fit by a more flexible approximation to the accounting identity than that given by the Cobb-Douglas. It is, therefore, not possible empirically to test the existence of the aggregate production function or the aggregate marginal productivity theory of factor pricing.

Source: Felipe and McCombie (2005a)

FOUR SIMULATION EXERCISES⁷

(i) *Fisher’s (1971) ‘Aggregate Production Function and the Explanation of Wages’*⁸

Fisher’s (1971) approach in his simulation experiments was to start with well-defined Cobb-Douglas micro-production functions at the firm or industry level. Having constructed the data for these separate firm production functions annually over a twenty-year period, the statistics were then summed and used to estimate an aggregate production function. A proxy for the aggregate capital stock was constructed, but this suffered from an aggregation problem. When the macroeconomic data were used to estimate an aggregate production function, Fisher, to his evident surprise, found the results were remarkably well determined and the data gave a good prediction of the wage rate, even though the aggregate production function did not exist.

To elaborate: Fisher proceeded by constructing a large number of hypothetical economies, each comprising of 2, 4, or 8 “firms”, depending upon the experiment. The micro Cobb-Douglas production functions of each firm exhibited constant returns to scale. Perfect competition was assumed to prevail. Hence, the underlying economy was quintessentially neoclassical. The individual firms had different output elasticities; in one series of experiments the values of labour’s output elasticities were chosen to be uniformly spread over the range of 0.7 to 0.8 and, in the other, over the range of 0.6 to

⁷ For reasons of space, we do not discuss the Monte-Carlo simulation experiments of Felipe and Holz (2001) that give some interesting insights into the econometric issues involved in the estimation of the Cobb-Douglas.

⁸ See also the discussion in Shaikh (1980)

0.9, so that in the four-firm case the values were 0.6, 0.7, 0.8, and 0.9. The unweighted average in all cases was 0.75.

The labour force and the capital stock were constructed to grow at predetermined rates over the 20-year period. Technical change occurred at a constant rate that differed between firms, or was absent. Output was homogeneous and capital was heterogeneous and firm specific. Given this latter constraint, labour was allocated between firms such that the marginal product of labour was constant across firms. The heterogeneous capital was *not* allocated between firms so that the marginal dollar invested in each firm was the same. Moreover, as the capital stocks were heterogeneous, they could not be simply added together, so an index, with all its attendant aggregation problems, had to be constructed.

Consequently, there were a number of reasons for anticipating that the aggregate Cobb-Douglas production function would not give a good fit to the generated data.

- The exponents of the individual Cobb-Douglas micro-productions differed.
- Capital was firm specific and not allocated optimally between firms.
- The heterogeneity of the capital stock meant that an index of capital has to be constructed, with the consequent aggregation problems.
- The firm data were summed arithmetically to give the aggregate variables.

Fisher ran 830 simulations using a number of different assumptions and estimated the following relationships using time-series data aggregated across the individual firms:

$$\ln V_t = c + b_4 t + b_5 \ln J_t^* + b_6 \ln L_t \quad (12)$$

$$\ln(V_t/L_t) = c + b_4 t + b_5 \ln (J_t^* / L_t) \quad (13)$$

where V is aggregate value added⁹ and J^* is an index of capital, which will be discussed below. Note that it differs from J used earlier in equation (6). (The time trend was dropped for the experiments where no technical change was introduced.)

⁹ Note that as output is assumed to be homogeneous by Fisher, we could equally have used the notation Q .

Fisher found uniformly high R^2 s of generally around 0.99, a value not untypical of R^2 s found using real, as opposed to hypothetical, data. Generally speaking, the aggregate production functions gave well-defined estimates, especially when constant returns were imposed to remove the multicollinearity between $\ln L$ and $\ln J^*$ (equation (13)).

However, the main focus of the study was on the degree to which the aggregated production function succeeded in explaining the generated wage data. It was found that, in the main, there were exceptionally good statistical fits, much to Fisher's surprise.

We should not expect the prediction of wages to be very accurate if the variance of labour's share is large, but "while it is thus obvious that a low variance of labor's share is a necessary condition for a good set of wage predictions, it is by no means obvious that this is also a sufficient condition. Yet, by and large, we find this to be the case" (Fisher 1971, p.314). This result occurs even when it can be shown unequivocally that the "underlying technical relationships do not look anything like an aggregate Cobb-Douglas (or indeed *any* aggregate production function) in any sense" (p.314, emphasis in the original). Fisher came to the conclusion that "the point of our results, however, is not that an aggregate Cobb-Douglas fails to work well when labor's share ceases to be roughly constant, it is that an aggregate Cobb-Douglas will continue to work well so long as labour's share continues to be roughly constant, *even though that rough constancy is not itself a consequence of the economy having a technology that is truly summarised by an aggregate Cobb-Douglas*" (Fisher, 1971, p.307, emphasis added).

Why did Fisher get such surprising results? We may explain this as follows.¹⁰ Consider n firms or industries, each of which has a "true" production function given by $Q_{it} = A_{it} K_{it}^{\alpha_i} L_{it}^{(1-\alpha_i)}$ where $i = 1, \dots, n$, and the output elasticities differ. K is the firm-specific capital stock (in terms, of say, numbers of identical machines). To generate an aggregate capital stock, Fisher notes that Euler's theorem holds:¹¹

$$V_t \equiv w_t L_t + \sum_{i=1}^n \rho_{it} K_{it} \quad (14)$$

¹⁰ See Shaikh (1980) for an explanation along different lines.

¹¹ Note that as equation (14) is an accounting identity, it will hold under all circumstances.

where ρ_t is again the rental price of capital, i.e., the competitive cost of hiring a machine for one period. “This means that at any moment of time, the sum of¹² the right-hand side of [14] makes an excellent capital index” (p. 308). Fisher therefore runs the model for the individual firms over the twenty-year period, and then obtains the sum of gross profits from the accounting identity for the firm. Then summing the number of machines for each firm, he obtains an average rental price of capital for each firm, which by definition is constant over the period:

$$\bar{\rho}_i \equiv \frac{\sum_{t=1}^{20} \rho_{it} K_{it}}{\sum_{t=1}^{20} K_{it}} \quad (15)$$

The index of the aggregate capital stock is then given by:

$$J_t^* \equiv \sum_{i=1}^n \bar{\rho}_i K_{it} \quad (16)$$

It should be noted that this index does not fulfil the necessary aggregation conditions.

The problem, of course, occurs because the relative magnitudes of the $[\rho_t(t)]$ not only do not remain constant over time but also are not independent of the magnitude of $L(t)$; this is the essence of the capital-aggregation problem.

Nevertheless, it seems clear that an aggregate production function will do best if its capital index comes as close as possible to weighting different capital goods by their rentals. (Fisher, p.308, omitting a footnote)

The definition of value added for the i^{th} firm is:

$$V_{it} \equiv w_{it} L_{it} + \rho_{it} K_{it} \equiv w_{it} L_{it} + \frac{\rho_{it}}{\bar{\rho}_i} J_{it}^* \quad (17)$$

¹² Fisher clearly means “on” here rather than “of”.

We may sum equation (17) over the n firms to give

$$V_t \equiv \sum_i^n V_{it} \equiv w_t L_t + \delta_t J_t^* \quad (18)$$

where w_t is the (weighted) average wage rate and $\delta_t \equiv (V_t - w_t L_t) / J_t^*$. The variable δ_t will be approximately equal to unity to the extent that the deviations of ρ_{it} from $\bar{\rho}_{it}$ tend to wash out when aggregated across firms. In other words, for every firm for which $\bar{\rho}_{it}$ overstates ρ_{it} there is a firm (or group of firms) where the ρ_{it} understates the rental price by approximately the same amount. A stronger assumption that gives the same result is that the rental price of capital for each firm does not greatly vary over time so $\rho_{it} \equiv \bar{\rho}_{it}$.¹³ It may be seen that the aggregate share of labour will be

$$(1 - a_t) = \sum_{i=1}^n (1 - a_i) \theta_{it} \quad \text{where } \theta_{it} = V_{it} / V_t \text{ and } a_i \text{ is constant over time. } (1 - a_t) \text{ will be}$$

constant if θ_{it} is assumed either to be roughly constant or to vary in such a way as to make $(1 - a_t)$ constant.¹⁴ We can now explain why an aggregate production function will give a good fit to the data. Even though the factor shares differ between firms, if in aggregate they are roughly constant, then assuming $\delta = 1$ or is constant over time, differentiating equation (18) and integrating will give

¹³ Equation (18) differs from the identity derived from the national accounts $V_t \equiv w_t L_t + r_t J_t$, where r_t is the rate of profit. J_t is the value of the capital stock calculated by the perpetual inventory method and equals the number of machines multiplied by their purchase price appropriately deflated (not their rental price, which is the price per period). As we demonstrated above, if we assume for expositional purposes that r_t equals the rate of interest, then $J_{it} = (\rho_{it} / r_{it}) K_{it}$ and $V_t \equiv w_t L_t + \delta_t J_t^* \equiv w_t L_t + r_t J_t$. (For expositional ease, we again abstract from capital gains and depreciation.) Consequently, if $\delta_t \equiv 1$ then $J_t^* \equiv r_t J$ or the total compensation of capital (*c.f.*, equation (14)).

¹⁴ With two firms, the firms' shares in total output have to be constant for aggregate labour's share (or the aggregate output elasticity of labour) to be constant. (This assumes that the individual firm's labour shares are constant.) But this is not true if there are more than two firms. Take the four-firm case where the labour shares are 0.6, 0.7, 0.8 and 0.9. At time t , if the firms' shares in total output are 0.25, 0.25, 0.25 and 0.25, the aggregate value of labour's share will be 0.75. It will, however, still take the same value at time $t+1$ if the firms' shares change to 0.167, 0.333, 0.333, and 0.167.

$$V_t = Bw_t^{(1-a)}J_t^*L_t^{(1-a)} \text{ or } V_t = A_0e^{\lambda t}J_t^*L_t^{(1-a)} \quad (19)$$

where λ is the constant growth rate of w_t weighted by $(1-a)$ and B is again the constant of integration. Thus, as Fisher (1971, p.325) concludes, it is “very plausible that in these experiments rough constancy of labor’s share should lead to a situation in which an aggregate Cobb-Douglas gives generally good results including good wage predictions, even though the underlying technical relationships are not consistent with the existence of any aggregate production function and even though there is considerable relative movement of the underlying firm variables”.

However, our interpretation is that the underlying micro-production functions will give constant firm-level factor shares for purely neoclassical reasons. It will be recalled that the firms are assumed to have Cobb-Douglas production functions which will give constant factor shares. Although the weights (the firms’ shares in total output) attached to them for aggregation may change over time, this does not prevent the shares from being roughly constant. Solow (1958) discusses why an aggregate factor share often shows less volatility than the individual shares that constitute it. Fisher himself does not find this explanation convincing (p.325, fn. 23),¹⁵ but it is hard to see what logically could be a more plausible explanation. Of course, it could be argued that if we are correct, the aggregate production function could be viewed as being a reasonable approximation for the underlying Cobb-Douglas technology, *pace* Fisher. We shall next turn to three simulations where this clearly is not the case.

(ii) The Evolutionary Growth Model of Nelson and Winter (1982)

The next example we shall consider is the evolutionary model of Nelson and Winter (1982, chapter 9). While, perhaps unnecessarily, conceding that the neoclassical approach to growth has served to give coherence to many individual research projects, Nelson and Winter (1982, p.206) nevertheless consider that “the weakness of the theoretical structure is that it provides a grossly inadequate vehicle for analysing

¹⁵ Fisher argues that in his simulations “relative outputs do not seem to be very constant”, but as we have seen in footnote 14, this is not necessary for aggregate labour’s share to be constant if the number of firms exceeds 2.

technical change”. What is particularly interesting is that they develop a model where individual firms have a *fixed-coefficients* production function and, as we shall see, their underlying behaviour is far from the usual neoclassical assumptions of the theory of the firm.

Their simulation model is one where a hypothetical economy is made up of a number of firms producing a homogeneous good. The technology available to each firm is, as we have said, one of fixed-coefficients, but with a large number of possible ways of producing the good given by different input coefficients (ϕ_L, ϕ_K) of differing efficiencies. However, the firm does not know the complete set of the input-output coefficients that are available to it, and so cannot immediately choose the best-practice technology. It only learns about the different techniques by engaging in a search procedure. The firms are not profit maximisers, but are satisficers and will only engage in such a search for a more efficient technique if the actual rate of profit falls below a certain satisfactory minimum, set at 16 percent.

There are two ways by which the firm may learn of other fixed-coefficients techniques. The first is the *innovation* process. The firm engages in a localised search in the input-coefficient space. This potentially comprises the complete set of possible existing techniques, but the firm will be only concerned with a particular subset. This is because it is assumed that the probability of a firm identifying a new technique is a declining function of the “distance” in terms of efficiency between any particular new technique and the firm’s existing technology. Consequently, the firm only searches locally in the input-coefficient space near its existing technique. The “distance” between the efficiency of a technique h' compared with the current technique h is a weighted average of $\ln(\phi_K^h / \phi_K^{h'})$ and $\ln(\phi_L^h / \phi_L^{h'})$ with the weights summing to unity.

Consequently, if the weight of $\ln(\phi_K^h / \phi_K^{h'})$ is greater than 0.5, the result will be that it is more difficult to find a given percentage reduction in the output-capital ratio than in the output-labour ratio. The converse is true if the weight is less than one-half.

Secondly, there is the *imitation* process where the firm discovers the existence of, and adopts, a more efficient technique because other firms are already using it. It is assumed that the probability of discovering this technique is positively related to the share of output produced by all the firms using this technique. This is similar to diffusion

models where a firm that is not using the current best-practice technique learns of it with an increasing probability as more and more firms adopt it.

The overall probability of a firm finding a new technique h' is modelled as a weighted average of the probability of finding the technique by local search and by imitation. The exact values of the weights chosen in calibrating the model will determine whether the firm engages in local search or in imitation. The firm will adopt h' only if it gives a higher rate of profit than that obtained by the existing technique, but it is also possible for the firm to misjudge the input coefficients of an alternative technique. The model is sufficiently flexible for new firms to appear.

The wage rate is endogenously determined by labour demand and supply conditions in each time period. The labour supply is constructed to grow at 1.25 per cent per annum. The prevailing wage rate affects the profitability of each firm, given the technique it is using. The behaviour of the industry as a whole also affects the wage rate. Each firm is assumed to always operate at full capacity, and so in effect Say's law operates and there is no lack of effective demand.

The simulations show that the increase in wages has the effect of moving firms towards techniques that are relatively capital intensive. As a firm checks the profitability of the technique when there is an increased wage rate, it will be the more capital-intensive techniques that will pass the test. While a rising wage rate will make all techniques less profitable, those that are labour-intensive will be more adversely affected. However, as Nelson and Winter (1982, p.227) point out, "while the explanation has a neoclassical ring, it is not based on neoclassical premises". The firms are not maximizing profits. "The observed constellations of inputs and outputs cannot be regarded as optimal in the Paretian sense: there are always better techniques not being used because they have not yet been found and [there are] always laggard firms using technologies less economical than current best practice."

The model was simulated with a view to comparing the outcome with Solow's (1957) results from fitting an aggregate production function to US data. To achieve this, the input-coefficient pairs space was derived from Solow's historical data – the US non-farm private business sector from 1909 to 1949. The simulation results produce industry data very similar to Solow's historical data. Indeed, if aggregate Cobb-Douglas production functions are fitted to Nelson and Winter's generated data, very good fits are obtained with the R^2 s often over 0.99 and the estimated aggregate "output elasticity with

respect to capital” (which, in fact, does not exist) often close to capital’s share, although there are one or two exceptions. As Nelson and Winter (1982, p. 226) observe, “the fact that there is no production function in the simulated economy is clearly no barrier to a high degree of success in using such a function to describe the aggregate series it generates.”

For our purposes, it is worth emphasizing that the simulated macroeconomic data suggests an economy characterized by factors being paid their marginal products and an elasticity of substitution of unity, even though we know that every firm is subject to a fixed-coefficients technology.¹⁶ The reason why the good fit to the Cobb-Douglas production function is found is once again because the factor shares produced by the simulation are relatively constant. Nelson and Winter (1982, p.227) summarise their findings as follows:

On our reading, at least, the neoclassical interpretation of long-run productivity change is sharply different from our own. It is based on a clean distinction between “moving along” an existing production function and shifting to a new one. In the evolutionary theory, substitution of the “search and selection” metaphor for the maximization and equilibrium metaphor, plus the assumption of the basic improbability of procedures, blurs the notion of a production function. In the simulation model discussed above, there was no production function – only a set of physically possible activities. The production function did not emerge from that set because it was not assumed that a particular subset of the possible techniques would be “known” at each particular time. The exploration of the set was treated as a historical, incremental process in which nonmarket information flows among firms played a major role and in which firms really “know” only one technique at a time.

(iii) Shaikh’s (2005) Non-Linear Goodwin Growth Model and the Cobb-Douglas Production Function

Shaikh (2005) provides further evidence of the difficulty of estimating an aggregate production function by elaborating on his 1987 entry in the *New Palgrave*. He generates hypothetical data by simulating a slightly modified version of the Goodwin (1967)

¹⁶ Houthakker (1955-56) shows that if firms have a fixed-coefficients technology and firm size is distributed as a Pareto distribution, then the aggregate production function will be a Cobb-Douglas with diminishing returns to scale.

growth model, which is based on a fixed-coefficients production function with Harrod-neutral technical change. However, as the data set has the property that factor shares are roughly constant, not surprisingly, he is able, eventually, with a judicious choice of a time path for technical change, to show that the Cobb-Douglas production function gives an excellent fit to the data. The regressions using the hypothetical data are also contrasted with those using actual data for the US economy over the postwar period. (The latter are from the Bureau of Economic Analysis's National Income and Product Accounts and associated wealth stocks.)

The simulation model may be described as follows. The level of output is given by a fixed-coefficients production function:

$$V = \min\left(\frac{L}{\phi_L(t)}, \frac{J}{\phi_K}\right) \quad (20)$$

where $\phi_L(t) = \phi_{L0}e^{-\lambda t}$. Consequently, over time, the amount of labour required to produce a given volume of output falls at the rate λ , or, what comes to the same thing, labour productivity increases at the rate λ , which is taken to be 2 per cent per annum. Thus, machines of more recent vintages require less labour, but the same amount of capital, as earlier machines. The capital coefficient (ϕ_K), however, is constant over time, so technical change is labour augmenting. It follows from the conditions of production that $\hat{V} - \hat{L} = \lambda$ and $\hat{V} - \hat{J} = 0$ and as \hat{L} is assumed to grow at 2 per cent per annum, output and capital grow in equilibrium at 4 per cent (recalling that λ equals 2 per cent). This assumes that the economy is moving along its warranted path. Thus, we have two of Kaldor's stylised facts, namely, a constant growth of labour productivity and a constant capital-output ratio.

Shaikh constructs a hypothetical data set generated by the Goodwin model. The growth of the real wage rate is determined by the employment ratio (the ratio of employment to the labour force) and labour's share and has nothing to do with the technical conditions of production (as in the marginal productivity theory of factor pricing). A property of the production function is that a change in the wage rate will not affect the choice of technique; all it will do is alter the distribution of income. The fact that we are dealing with a fixed-coefficients technology means that the marginal

products cannot be defined. As Shaikh (2005, p. 451, italics in the original) emphasises, “it follows that the technological structure of this control group [Goodwin] model is entirely distinct from that of neoclassical production theory and associated marginal productivity rules”.

In steady-state growth, the parameters of the real wage growth function are such that the growth of the real wage is 2 per cent per annum, i.e., equal to the growth of labour productivity and this means that labour’s (and, hence, capital’s share) is constant. The model is stable in that after a shock, the growth of output converges to 4 percent per annum and labour’s share to a constant (approximately 0.84) and the employment ratio to a steady 95 per cent. Consequently, the simulated data series, like the actual US data, have factor shares that do not vary greatly over time. Nevertheless when a Cobb-Douglas is estimated with a linear time trend (in the log-level specification) or with a constant intercept (in the growth rate form), the results are poor regardless of whether the simulated or the actual US data are used and whether the Cobb-Douglas is freely estimated or has constant returns to scale imposed on the coefficients.

The reason is that notwithstanding the constancy of the factor shares, if the growth of the weighted wage rate and profit rate is not sufficiently constant, this can lead to poorly determined and biased coefficients of the factor inputs. In fact, both data sets show a pronounced fluctuation in the rate of profit, which has generally been found to be the main cause of other poor fits of the Cobb-Douglas (the wage rate is not so volatile around its trend). Shaikh notes that the *Solow Residual* is nothing other than the weighted average of the growth of the wage rate and the rate of profit, so that

$\hat{A}_t = a_t \hat{r}_t + (1 - a) \hat{w}_t$ and, if factor shares are constant, $A_t = B_0 r_t^a w_t^{(1-a)}$. Consequently, the only difference between the Cobb-Douglas and the identity is the restriction usually imposed on the Cobb-Douglas that the weighted growth of the wage rate and rate of profit is a linear function of time with a random error term. (If shares are not exactly constant over time, then this will provide another difference.) But even in the neoclassical schema, there is no reason why this should be the case. The actual time path of A_t can be approximated to any required degree of precision by a complex time trend such as a Fourier series. Shaikh further notes that if one wishes to use a smooth path of technical change, then it is always possible to construct a series $\hat{F}_t = \psi \hat{A}_t$ where, if $\psi < 1$,

this dampens, or smoothes, the fluctuations.¹⁷ Defining \hat{A}_t as $(\hat{V}_t - \hat{L}_t) - a(\hat{J}_t - \hat{L}_t)$ and taking ψ as either 0.2 or 0.6, Shaikh, not surprisingly, gets a very good fit to the data with the estimated coefficients of the inputs almost precisely the same as the factor shares.

(iv) Felipe and McCombie's (2006) Simulations: 'The Tyranny of the Accounting Identity'

Fisher (1971, p.325) concluded his paper with the remarks, which could equally be the conclusions of the other two simulation studies, that “the suggestion is clear, however, that labor’s share is not roughly constant because the diverse technical relationships of modern economies are truly representable by an aggregate Cobb-Douglas but rather that such relationships appear to be representable by an aggregate Cobb-Douglas *because* labor’s share happens to be roughly constant. If this is so, then the reason for such constancy becomes an important subject for further research” (emphasis in the original).

This was one of the starting points of Felipe and McCombie’s (2006)¹⁸ simulations. A major difference between their explanation and the others is that Felipe and McCombie draw an explicit and important distinction between a micro-production function, which is an engineering relationship, with output and capital measured in physical terms and the aggregate production function where they are measured in constant-price monetary terms. Consequently, some set of base-year prices has to be used to construct a constant-price monetary measure of output and capital to allow aggregation.

Felipe and McCombie adopted an approach different from those discussed above, in that they constructed two types of data for the firm. They postulated that there were well-defined firm micro-production functions, with output and the capital stock specified in physical terms, as ideally they should be. These micro-production functions were Cobb-Douglas, but the output elasticity of capital was deliberately chosen to be

¹⁷ Shaikh uses the notation F_t instead of \hat{F}_t and also allows its mean to differ from that of \hat{A}_t .

¹⁸ See also McCombie (2001).

0.75 and of labour, 0.25. This stands in marked contrast to the usual values found of 0.25 and 0.75, respectively. Then they constructed constant-price data for output for firm i using a mark-up pricing model:

$$p_i = (1 + \mu)wL_i / Q_i \quad (21)$$

where p is the price (£ per unit output), μ is the mark-up, taken as 0.333, and w is the exogenously given money wage rate which was assumed to be the same for each firm. The profit rate r took a value of 0.10 for each firm. The *value* of the capital stock was calculated residually through the accounting identity as $J_i \equiv (V_i - wL_i) / r$, where V_i is value added, constructed as $V_i = p_i Q_i$ by using equation (21) for each firm. The values of the factor shares are directly calculated using these value data. Labor's share is calculated as $a_i = (wL_i / V_i)$ and capital's share as $(1 - a_i)$. It should also be noted that $a_i = 1 / (1 + \mu)$, and so it takes a value of 0.75 for each firm, with a small variation due to an added random variable to prevent perfect multicollinearity. The researcher is assumed to know only the value data, i.e. V and J and not Q and K . Using these data and running a cross-firm regression gives:¹⁹

$$\begin{aligned} \ln V &= 2.867 + 0.250 \ln J + 0.750 \ln L & \bar{R}^2 &= 0.999 \\ &(478.77) \quad (45.41) \quad (136.40) & \text{s.e.r.} &= 0.0025 \end{aligned}$$

Consequently, it can be seen that the estimated output elasticity of labour is 0.75 (and not the 'true' value of 0.25) and of capital is 0.25 (and not 0.75).

Indeed, it is the constant mark-up that is solely responsible for generating the very good fit to the "spurious" Cobb-Douglas. To demonstrate this, the physical values of the three series Q , L and K were next generated as *random numbers*. V and J were calculated as before. Nevertheless, the estimation yielded a very good fit to the Cobb-Douglas with the values of the "output elasticities" the same as before. This does not necessarily mean that Felipe and McCombie are postulating that output is actually a random function of factor inputs. However, when one considers the complex production

¹⁹ The goodness of fit is determined by the random variable introduced into the construction of the value data to prevent perfect multicollinearity.

processes of any modern firm, there may be some individual parts of the process subject to fixed coefficients, whereas others are subject to differing elasticities of substitution, to say nothing of differences between plants in managerial and technical efficiencies. Thus, the “randomness” may simply be a reflection of the severe misspecification error inherent in specifying the micro-production function as a Cobb-Douglas. But the important point to note is that even in this case, where there is no well defined micro-production function, the use value-added data will give the impression that there exists a well-behaved aggregate Cobb-Douglas production function.

When the true micro-production functions exhibit strong increasing returns (the degree of homogeneity was set equal to 1.20), but the value of the mark-up is the same as before, estimating the unrestricted Cobb-Douglas production function gives a result that is virtually identical to that for constant returns to scale, and reported above, except for a change in the value of the intercept. This shows that even when there are increasing returns to scale at the micro level, using value data will mean that this is captured in the “level of technology” of the aggregate production function and estimates of the latter will suggest constant returns to scale.

Felipe and McCombie also used these hypothetical data to calculate the growth of total factor productivity (or the size of the Solow residual) for an industry which consisted of ten firms. It was assumed that each firm experiences the same rate of technical progress of 0.5 per cent per annum. The output elasticities in physical terms were the same as before, as was the mark-up.

As the rate of technical progress was the same for each firm, we can talk about the rate of technical progress being 0.5 per cent per annum; even in the case where we assume that the physical outputs of the various firms are not homogeneous. The values of the individual firm’s value added, constant price capital stock and employment were summed to give the industry values.

However, it was again assumed that all that can be used in empirical work, as is usually true in practice, is the constant-price value of output and of the capital stock. The growth of total factor productivity is given by:

$$TFPG \equiv \hat{V} - a\hat{L} - (1-a)\hat{J} \quad (22)$$

where now the shares of capital and labour are 0.75 and 0.25, respectively.

The rate of total factor productivity growth obtained by using the aggregated value data of the 10 firms and equation (22) came to 1.48 per cent per annum. The reason for the marked difference between these values and the “true” rate of technical progress of 0.5 per cent per annum is that labour’s share of output in value terms is 0.75, while the “true” output elasticity of the firms’ production functions is 0.25. Consequently, the true rate of technical progress cannot be determined using constant-price monetary values, as is the universal practice.

CONCLUSIONS

Fisher (1971, p.305) noted that Solow once remarked to him that, “had Douglas found labor’s share to be 25 per cent and capital’s 75 per cent instead of the other way around, we would not now be discussing aggregate production functions”. In this paper, we have shown that Douglas, by using monetary values in his estimations of the aggregate production function could not have failed to have found this result. Indeed, with knowledge of Kaldor’s stylized facts and the accounting identity linking total value added to the sum of wages and profits, we can predict the results of estimating various production functions before a single regression has been run. This has been shown, for example, by Felipe and McCombie (2005b) in the case of Mankiw, Romer and Weil’s (1992) well-known study, which actually tells us nothing we did not already know. It certainly cannot be interpreted as a test of the factors that determine economic growth or of the augmented Solow model.

Our nihilistic conclusion is that because theoretically the aggregate production function does not exist, and empirically it cannot be meaningfully estimated, it can shed no light on how real economies work. Consequently, neoclassical growth theory, which relies on the aggregate production function, can shed little, if any light, on “why growth rates differ”. We have also shown in section (iv) above how the concept of total factor productivity growth (or multifactor productivity growth as it is sometimes called) is equally flawed, even though it is now widely used by such bodies as the OECD as a well-established and accepted measure of productivity growth.

REFERENCES

Barro, R.J and Sala-i-Martin, X. (2004), *Economic Growth*, Cambridge MA: MIT press (2nd edition).

Bronfenbrenner, M. (1944), "Production Functions: Cobb-Douglas, Interfirm, Intrafirm", *Econometrica*. vol. 12, pp. 35-44.

Cohen, A. J. and Harcourt, G.C. (2003a), "Whatever Happened to the Capital Controversies?" *Journal of Economic Perspectives*, vol. 17, pp. 199-214.

Cohen, A. and Harcourt, G.C. (2003b), [Cambridge Capital Controversies] Response. *The Journal of Economic Perspectives*, vol. 17, pp. 227-232

Cramer, J.S. (1969) *Empirical Econometrics*, Amsterdam: North-Holland.

Douglas, P.H. (1948), "Are There Laws of Production?" *American Economic Review*, vol.38, pp. 1-41.

Douglas, P. H. (1976), "The Cobb-Douglas Production Function Once Again: Its History, Its Testing, and Some New Empirical Values", *Journal of Political Economy*, vol. 84, pp. 903-915.

Felipe, J. (2001a), "Endogenous Growth, Increasing Returns, and Externalities: An Alternative Interpretation of the Evidence", *Metroeconomica*, vol.52, pp. 391-427.

Felipe, J. (2001b), "Aggregate Production Functions and the Measurement of Infrastructure Productivity: A Reassessment", *Eastern Economic Journal*, vol. 27, pp. 323-344.

Felipe, J., and Adams, F.G. (2005), " 'A Theory of Production'. The Estimation of the Cobb-Douglas Function: A Retrospective View, *Eastern Economic Journal*, vol.31, pp. 427-445.

Felipe, J. and Fisher, F.M. (2003), "Aggregation in Production Functions: What Applied Economists Should Know", *Metroeconomica*, vol.54, pp. 208-262.

Felipe, J., and Holz, C. (2001), "Why do Aggregate Production Functions Work? Fisher's Simulations, Shaikh's Identity, and Some New Results", *International Review of Applied Economics*, vol. 15, pp. 261-285.

Felipe, J., and McCombie, J.S.L. (2001), "The CES Production Function, the Accounting Identity, and Occam's Razor", *Applied Economics*, vol. 33. pp.1221-1232.

Felipe, J., and McCombie, J.S.L. (2003), Methodological Problems with Neoclassical Analyses of the East Asian Miracle, *Cambridge Journal of Economics*, vol. 54, pp.695-721.

- Felipe, J. and McCombie, J.S.L. (2005a): “How Sound are the Foundations of the Aggregate Production Function”, *Eastern Economic Journal*, vol. 31, pp. 467-488.
- Felipe, J. and McCombie, J.S.L. (2005b), “Why are Some Countries Richer than Others? A Sceptical View of Mankiw-Romer-Weil’s Test of the Neoclassical Growth Model.” *Metroeconomica*, vol.56 pp.360-392.
- Felipe, J. and McCombie, J.S.L. (2006), “The Tyranny of the Accounting Identity: Growth Accounting Revisited”, *International Review of Applied Economics*, vol.20, pp. 283-299.
- Felipe, J. and McCombie, J.S.L. (2007), “On the Rental Price of Capital and the Profit Rate: The Perils and Pitfalls of Total Factor Productivity Growth”, *Review of Political Economy*, vol.19, pp. 317-345.
- Ferguson, C.E. (1971), “Capital Theory Up to Date: A Comment on Mrs. Robinson’s Article”, *Canadian Journal of Economics*, vol. IV, pp.250-254.
- Fisher, F.M. (1971), “Aggregate Production Functions and the Explanation of Wages: A Simulation Experiment”, *Review of Economics and Statistics*, vol. 53, pp. 305-325.
- Fisher, F.M. (1992) , *Aggregation. Aggregate Production Functions and Related Topics*, edited by J. Monz, London: Harvester Wheatsheaf.
- Fisher, F.M (2003), Comment on Cohen and Harcourt, *Journal of Economic Perspectives*, vol. 17, pp. 227-232.
- Fisher, F.M. (2005), “Aggregate Production Functions – A Pervasive, but Unpersuasive, Fairytale”, *Eastern Economic Journal*, vol.31, pp.489-491.
- Friedman, M. (1951), “The Methodology of Positive Economics”, in Friedman, M. (ed.) *Essays in Positive Economics*, Chicago: Chicago University Press.
- Goodwin, R.M. (1967) “A Growth Cycle” in Feinstein C.H. (ed.) *Socialism, Capitalism and Economic Growth, Essays Presented to Maurice Dobb*, Cambridge: Cambridge University Press.
- Houthakker, H.S. (1955-1956), “The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis”, *Review of Economic Studies*, vol.23, pp.27-31.
- Intriligator, M.D. (1978), *Econometric Models, Techniques and Applications*, Englewood-Cliffs, NJ: Prentice Hall.
- McCombie J.S.L. (1987) Does the Aggregate Production Function Imply Anything about the Laws of Production? A Note on the Simon and Shaikh Critiques. *Applied Economics*, vol. 19, pp. 1121-1136.

- McCombie, J.S.L. (1998a), “ ‘Are There Laws of Production?’: An Assessment of the Early Criticisms of the Cobb-Douglas Production Function”, *Review of Political Economy*, vol. 10, pp. 141-173.
- McCombie, J.S.L. (1998b), Paradigms, Rhetoric, and the Relevance of the Aggregate Production Function, in Arestis, P. (ed.), *Method, Theory and Policy in Keynes. Essays in Honour of Paul Davidson*, vol.III, Aldershot: Edward Elgar.
- McCombie, J.S.L. (2000-2001), “The Solow Residual, Technical Change and Aggregate Production Functions”, *Journal of Post Keynesian Economics* vol.23, pp. 267-297. (Errata, vol. 23(3), p.544.)
- McCombie, J.S.L. (2001) “What do Aggregate Production Functions Show? Second Thoughts on Solow’s ‘Second Thoughts on Growth Theory’ ”, *Journal of Post Keynesian Economics*, vol. 23, pp. 589-615
- McCombie, J.S.L. & Dixon, R. (1991), “Estimating Technical Change in Aggregate Production Functions: A Critique”, *International Review of Applied Economics*, vol. 5, pp. 24-46.
- McCombie, J.S.L. and Thirlwall, A.P. (1994), *Economic Growth and the Balance-of-Payments Constraint*, Basingstoke: Macmillan.
- Mankiw, N.G., Romer, D., and Weil, D.N. (1992), “A Contribution to the Empirics of Economic Growth”, *Quarterly Journal of Economics*, vol. 107, pp. 407-437.
- Marshall J. and Andrews, W.H. (1944), “Random Simultaneous Equations and the Theory of Production”, *Econometrica*, vol. 12, pp.143-205.
- Nelson R.R and Winter S.G. (1982) *An Evolutionary Theory of Economic Change*. Cambridge MA: Harvard University Press.
- Phelps Brown E.H. (1957), "The Meaning of the Fitted Cobb-Douglas Function", - *Quarterly Journal of Economics*, vol.71, pp.546-560.
- Samuelson, P. A. (1979), “Paul Douglas’s Measurement of Production Functions and Marginal Productivities”, *Journal of Political Economy*, vol. 87, pp. 923-939.
- Shaikh, A. (1974), “Laws of Production and Laws of Algebra, the Humbug Production Function, *Review of Economics and Statistics*, vol.56, pp. 115-120.
- Shaikh, A. (1980) "Laws of Production and Laws of Algebra: Humbug II", in Nell, E.J. (ed.) *Growth, Profits and Property*, Cambridge: Cambridge University Press.
- Shaikh, A. (1987) Humbug Production Function in Eatwell, J., Milgate, M. and Newman, P. (eds) *The New Palgrave. A Dictionary of Economic Theory and Doctrine*. London: Macmillan.
- Shaikh, A. (2005) “Nonlinear Dynamics and Pseudo-Production Functions”, *Metroeconomica*, vol. 31, 447-466.

- Simon, H. A. (1979a), "On Parsimonious Explanations of Production Relations", *Scandinavian Journal of Economics*, vol.81, pp. 459-474.
- Simon, H.A. (1979b), "Rational Decision-Making in Business Organizations", *American Economic Review*, vol 69, pp.493-513. (Nobel Memorial Lecture, 8 December, 1997.)
- Simon, H. A. and Levy F.K. (1963), "A Note on the Cobb-Douglas Function", *Review of Economic Studies*, vol. 30, pp. 93-94.
- Solow, R. M. (1957), "Technical Change and the Aggregate Production Function", *Review of Economics and Statistics*, vol. 39, pp. 312-320.
- Solow, R.M. (1958), "A Skeptical Note on the Constancy of Relative Shares", *American Economic Review*, vol. 48, pp.618-663.
- Solow, R.M. (1974) "Laws of Production and Laws of Algebra: The Humbug Production Function: A Comment", *Review of Economics and Statistics*, vol.56, p.121.
- Solow, R.M. (1987), "Second Thoughts on Growth Theory", in Steinherr, A. and Weiserbs, D. (eds), *Employment and Growth: Issues for the 1980s*, Dordrecht: Martinus Nijhoff Publishers.
- Wallis, K.F. (1979), *Topics in Applied Econometrics*. London: Gray-Mills Publishing,
- Walters, A.A. (1963), "Production and Cost Functions", *Econometrica*, vol.31, pp.1-66.